

## Effect of a Pressure Ridge on Ice-Coupled Gravity Waves

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**The boundary value problem associated with scattering of ice-coupled gravity waves by a thin vertical ice sheet in infinitely deep water is explicitly solved. The method of solution utilizes a connection that helps to decompose the problem into two relatively easy problems. The decomposed problems are explicitly solved by the aid of a weakly singular integral equation. Reflection coefficients are computed in both cases of ice sheet with and without compressive force in it. The model predicts a few resonant frequencies of ice-coupled waves at which complete reflection occurs. Further, scattering quantities are found to be sensitive to the ice sheet parameters.**

### INTRODUCTION

Flexural or ice-coupled gravity waves are created when free surface gravity waves propagate through flexible floating structures such as ice sheets and elastic sheets. They also arise in floating platforms such as very large floating structures (VLFS) whose hydrodynamic and hydroelastic behaviour is important when utilizing ocean space for activities and development. The development of these floating structures is reviewed by Kashiwagi (2000). A similar review was done by Squire et al. (1995) in the context of wave interactions with an ice-covered surface. Many mathematical techniques have been developed to study the interaction of waves with these floating ice-covered structures. Eigenfunction expansions, integral equation methods, approximate methods, Fourier transform methods, and Wiener–Hopf techniques are some of the popular methods in the literature; see Sahoo et al. (2001), Meylan (1997), and Balmforth and Craster (1999), to name a few.

Many interesting studies have been done on flexural gravity wave interactions with natural abnormalities within the floating structures (for example, cracks and variable thicknesses in the ice sheets). One such study describes the effect of cracking in the ice-cover on ice-coupled or flexural gravity waves. Marchenko (1993) solved the diffraction problems involving single or many cracks. Squire and Dixon (2000) provided an analytical solution to the problem by making use of a Green function technique. The variational formulation has been utilized by Marchenko (1999) to study the problem and to discuss the edge waves. Evans and Porter (2003) analysed the oblique wave scattering caused by a narrow crack in an ice sheet floating on water of finite depth by using a Green function method and the eigenfunction expansion method. Porter and Evans (2014) found the existence of trapped modes in the presence of a crack existing in a three-dimensional infinite elastic plate.

Pressure ridges are naturally formed as variations in floating ice sheets, and they are mostly formed by the interaction of floating ice sheets. They also form due to the stresses within the plane of the ice sheet. The upper part of the ridge is called the sail and the lower part of the ridge is called the keel. A kinematic model for the formation of a pressure ridge was provided by Permerter

and Coon (1972). The problem of finding natural vibrations of a hummock ridge attached with a floating elastic ice sheet was analysed by Marchenko (1995). Williams and Squire (2004) described a theoretical analysis of wave propagation beneath an inhomogeneous sea ice with the incorporation of pressure ridges as local variations in the ice thickness. They obtained the solution by using a finite-depth Green function approach. Along similar lines, Bennetts et al. (2007) have solved the problem of the scattering of water waves involving a floating ice sheet of varying thicknesses by making use of multi-mode expansion. Bennetts and Squire (2012) have investigated the sensitivity of the attenuation created by inhomogeneities in the ice cover such as floes, cracks, and pressure ridges through scattering, using an established model. Meng and Lu (2017) and the references therein discussed similar flexural gravity wave interactions in the context of the design of VLFS.

In the present paper, the fully formed pressure ridge has been approximately modelled as a piercing thin elastic vertical barrier of finite length through the floating ice cover. Then, the effect of the sail or the upper part of the pressure ridge in the floating ice sheet on the ice-coupled gravity waves may be negligible. Meylan (1995) first modelled the ice-coupled ocean wave scattering problem involving ice keels. However, Meylan (1995) ignored the floating ice sheet at the surface while solving the problem mathematically; perhaps, this choice was due to the non-availability of methods of solution for the realistic problem in which both the floating ice sheet and the vertical ice sheets are involved. Meylan (1995) undertook only a primitive model of the actual physical problem, that is, the scattering problem of free surface gravity waves involving a vertical ice sheet. The aim of the present work is to find the effect of the ice keel on the propagating gravity waves in the floating ice sheet, since the pressure ridge is approximately and ideally modelled here as a finite-length surface-piercing vertical barrier. The scattering problem for the realistic pressure ridges can only be handled semi-analytically or numerically, since both the floating and the vertical ice sheets are of variable thickness. Here, an explicit method of solution has been demonstrated for the linearized ice-coupled gravity wave scattering problem involving a vertical ice sheet. The associated mixed boundary value problem has been handled for an analytical solution, and the numerical results have been obtained for the scattering quantities. The problem has been reduced to solve a dual integral equation with a kernel comprised of trigonometric functions. Note that the crucial factor in the reduction process has been the utilization of expansion formulae developed by Manam

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