

Fundamental Investigation on the Applicability of a Higher-Order Consistent ISPH Method

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The paper presents a high-order consistent incompressible smoothed particle hydrodynamics (ISPH) fluid model for accurate simulation of ocean engineering problems. The high-order consistent discretization schemes on differential operators are derived through consideration of Taylor-series expansion up to second-order differential terms. The derived consistent discretization schemes are applied to the calculations of a Laplacian term of pressure in the Poisson pressure equation (PPE) and pressure gradient term in the momentum equation. To enhance and ensure the accuracy, stability, and conservation property of the model, the enhanced schemes developed by our research team—namely, the higher-order source term, error-compensating source term, and dynamic stabilizer schemes—are also incorporated. The proposed high-order consistent ISPH fluid model is validated through reproduction of a set of numerical examples.

INTRODUCTION

Lagrangian meshfree computational methods, or the so-called particle methods such as smoothed particle hydrodynamics (SPH; Gingold and Monaghan, 1977) or incompressible SPH (ISPH; Shao and Lo, 2003), have recently been attracting a lot of interest in various engineering fields. Thanks to their Lagrangian meshfree description of motion, the particle methods possess great advantages—for example, being free from calculation of an advection term and stable/natural tracking of complex moving boundaries. In the field of ocean engineering, one can easily find a number of existing applications of particle methods toward violent free-surface fluid flow and its interaction with rigid/deformable structures, as comprehensively reviewed in Luo et al. (2021) and Gotoh et al. (2021).

In the SPH method, consistency has been considered one of the key challenges (SPHERIC Grand Challenges; Vacondio et al., 2021), and much effort has been devoted to enhancement of consistency of particle-based differential operator discretization schemes (e.g., Fatehi and Manzari, 2011; Khayyer and Gotoh, 2011; Duan et al., 2021). The application of high-order consistent discretization schemes for differential operators would reduce the so-called discretization error (Quinlan et al., 2006) and result in enhancement of accuracy.

In the context of the ISPH method, first-order consistency correction on the pressure gradient model has been widely used, and its enhanced effects have been presented in the literature (e.g., Lind et al., 2012; Khayyer et al., 2017b; Zheng et al., 2017). On the other hand, there have been few ISPH studies on the development of second-order operator schemes or high-order Laplacian ones, as well as very few works on applications of high-order schemes for ISPH fluid simulations. The work of Zhang et al. (2021) is one of the few examples of application of the high-order SPH Laplacian model (a recently developed quadric

semi-analytical finite-difference interpolation (QSFDI) scheme; Yan et al., 2020) to the ISPH fluid model, and to the best of our knowledge, there have been no studies regarding the application of a second-order gradient discretization operator scheme into an ISPH fluid model, although these topics have been attracting a lot of interest in other branches of projection-based particle methods: the moving-particle semi-implicit (MPS) method and the consistent particle method (e.g., Gao et al., 2021; Luo et al., 2022).

In light of this background, this work aims at the development of a high-order consistent ISPH fluid model for reliable simulation of incompressible free-surface fluid flow. The SPH-based derivation on high-order consistent discretization schemes for differential operators is performed with consideration of Taylor-series expansion. The obtained consistent discretization schemes are adopted to the approximations of the Laplacian term of pressure in a Poisson pressure equation (PPE) and pressure gradient term in the momentum equation. The ISPH fluid model is further enhanced by incorporating several existing refined schemes. The performance of the proposed ISPH fluid model is validated through reproductions of ocean engineering numerical examples.

NUMERICAL METHOD

Enhanced Incompressible SPH Method

In the ISPH method, the motion of a fluid particle is governed by the continuity and Navier–Stokes equations:

$$\frac{D\rho}{Dt} + \rho \nabla \cdot \mathbf{u} = 0 \quad (1)$$

$$\frac{D\mathbf{u}}{Dt} = -\frac{\nabla p}{\rho} + \nu \nabla^2 \mathbf{u} + \mathbf{g} \quad (2)$$

where D/Dt indicates the Lagrangian time derivative, \mathbf{u} is a velocity vector, t represents time, p refers to pressure, ρ signifies fluid density, ν denotes the kinematic viscosity, and \mathbf{g} denotes the gravitational acceleration vector.

These principal equations (Eqs. 1 and 2) lead to the PPE based on the Chorin's projection method (Chorin, 1968):

$$\left\langle \frac{\Delta t}{\rho} \nabla^2 p^{k+1} \right\rangle_i = \frac{1}{\rho_0} \left(\frac{D\rho}{Dt} \right)_i^c - S_{\text{ECS}}; \quad \rho_i = \sum_j m_j w_{ij} \quad (3)$$

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KEY WORDS: Particle method, high-order model, consistency, smoothed particle hydrodynamics, projection method, accuracy, energy conservation.