

# Comparative Analysis of Three Smoothed Particle Hydrodynamics Methods in Modeling Free-surface Flows

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Smoothed particle hydrodynamics (SPH) has been extensively applied in ocean/coastal engineering. In general, the SPH method can be divided into weakly compressible SPH (WCSPH) and incompressible SPH. The former is used more often than the latter because of its higher computational efficiency. The standard weakly compressible SPH has the issue of pressure fluctuations. Two typical WCSPH models developed to eliminate pressure fluctuations are the  $\delta$ -SPH and Riemann-SPH models. The  $\delta$ -SPH model introduces a diffusive term in the continuity equation, and Riemann-SPH uses a Riemann solver that determines the interaction between particles by a simple limiter to decrease the inherent numerical dissipation. In the present work, three models (standard-WCSPH,  $\delta$ -SPH, and Riemann-SPH) are first considered to simulate the case of an oscillating drop as a classical free-surface flow benchmark. A comparison between the numerical simulation and theoretical solution on the time variation of energy and semimajor axis is conducted. The detailed pressure field is also discussed. Then test cases of violent sloshing and standing waves are conducted to further compare the three models. The numerical results are compared with available analytical and experimental data.

## INTRODUCTION

Smoothed particle hydrodynamics (SPH) (Monaghan, 1992) is a mesh-free Lagrangian method that was first developed for studying astrophysics problems by Gingold and Monaghan (1977) and Lucy (1977), and it became very popular in simulating fluid flow because of its flexibility in adapting to complex geometries and describing free-surface flow (Liu and Liu, 2003; Gotoh and Khayyer, 2016; Luo et al., 2021). In general, the SPH method can be categorized into two different frameworks: weakly compressible SPH (WCSPH) and incompressible SPH (ISPH). The weakly compressible SPH limits fluid compressibility by imposing a large speed of sound to the equation of state (Monaghan, 1994), whereas ISPH (Shao and Lo, 2003; Zheng et al., 2014) achieves fluid incompressibility by solving a pressure Poisson equation. The former is often used because of its faster computational time when compared with the latter.

The early versions of WCSPH have had challenges in producing smooth pressure results (Violeau and Rogers, 2016). Many modifications to WCSPH have been suggested to reduce pressure fluctuations. Artificial viscosity is proposed by Monaghan and Gingold (1983) to eliminate pressure oscillations but may lead to

energy dissipation. One strategy is to add the diffusive term in the continuity equation to reduce pressure fluctuations ( $\delta$ -SPH). Molteni and Colagrossi (2009) proposed a numerical diffusive term inside the continuity equation, but this term was not compatible with the hydrostatic solution. Then Antuono et al. (2010) corrected the diffusive term of Molteni and Colagrossi (2009), and their term gave good results. Another strategy to reduce pressure fluctuations is using a Riemann solver, which determines the interaction between particles by a simple limiter to decrease the inherent numerical dissipation. The introduction of the Riemann solver led to the development of the SPH scheme called Riemann-SPH (Zhang et al., 2017a; Rezavand et al., 2020). Zhang et al. (2017a) noted that the classic Riemann solver is generally more dissipative than those with artificial viscosity and developed a low-dissipation Riemann solver based on the WCSPH method. Green et al. (2019) derived that  $\delta$ -SPH formulation is equivalent to the adoption of the Riemann solver in the continuity equation.

In this paper, an open-source code, SPHinXsys (Zhang, Rezavand, Zhu et al., 2020), is applied to simulate free-surface flow. SPHinXsys, which is a multiphysics library based on smoothed particle hydrodynamics and Simbody, which provides a high-performance multibody physics object-oriented C++ application programming interface, was developed by Professor Xiangyu Hu and his colleagues at Technische Universität München for simulating fluid dynamic (Adami et al., 2012, 2013; Zhang et al., 2017a, 2017b), solid dynamic, and fluid-structure interactions (Rezavand et al., 2020; Zhang, Wei et al., 2020). The SPHinXsys model adopts the low-dissipation Riemann solver based on weakly compressible SPH for free-surface flow (Zhang, Wei et al., 2020). The aim of this study is to compare three SPH models (i.e., Standard-WCSPH,  $\delta$ -SPH, and Riemann-SPH), focusing on the aspects of

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