

# Parameterisation of Radiation Forces for Multiple Degree-of-Freedom Wave Energy Converters Using Moment-Matching

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**The motion of a wave energy converter (WEC) can be described in terms of an integro-differential equation, which involves a convolution operator. This convolution term accounts for the effect of radiation forces acting on the device and represents a computational and representational drawback both for simulation and analysis/design of control/estimation strategies. We present herein a moment-based strategy to compute a parametric form of the radiation force subsystem for multiple degree-of-freedom WECs. The strategy allows for the computation of a model that exactly matches the steady-state behaviour of the target system at a set of user-defined frequencies, while retaining the underlying physical properties of radiation forces. The potential and capabilities of the presented method are illustrated considering a CorPower-like device (heaving point absorber) as an application case.**

## INTRODUCTION

Among the different modelling approaches adopted in the wave energy literature (see Li and Yu, 2012), the speed with which numerical simulation may be performed makes the widely known boundary element method (BEM) a common choice to compute hydrodynamic parameters for a given wave energy converter (WEC) (Penalba et al., 2017). However, one of the major drawbacks of the BEM is that the results are computed in the frequency domain and, hence, can only characterise the steady-state motion of the WEC under analysis. Seeking a more comprehensive approach and following the well-known theory developed in Cummins (1962), the motion of a WEC can be expressed, in the time domain, using a particular well-known integro-differential equation of the convolution class. The presence of these convolution terms accounts for the effect of radiation forces acting on each of the different degrees of freedom (DoF) of the device, constituting a (hydrodynamic) coupling between these modes of motion.

The existence of these convolution terms represents a significant drawback both for motion simulation and for modern analysis/design of control/estimation strategies. From a motion simulation point of view, it is well known that the explicit computation of the convolution operator is computationally inefficient, often worsened by the necessity of a small (time) discretisation step to obtain accurate numerical integration. From a control/estimation theory point of view, the presence of these convolution mappings complicates the application of well-established results in the field, since modern control/estimation techniques are based on the availability of a state-space representation (at least in local coordinates) of the system under analysis (Faedo et al., 2017). Motivated by these drawbacks, researchers often seek for a parametric approximation of this radiation force subsystem in terms of a linear time-invariant dynamic representation, making explicit use

of the corresponding hydrodynamic characteristics of the device obtained from BEM solvers.

To be precise, the prevailing approach is to approximate each convolution term independently (see, for example, Pérez and Fossen (2008) and Giorgi and Ringwood (2019), as a single-input single-output (SISO) dynamic system, although the problem is inherently multiple-input multiple-output (MIMO), as a consequence of the multi-DoF characteristic of the WEC. One main disadvantage of this “multi-SISO” approach is that treating each convolution term independently often leads to an unnecessary high-order dimensional parameterisation of the radiation force subsystem, potentially rendering any control/estimation strategy challenging for real-time applications (Faedo et al., 2017).

We have recently presented a moment-matching-based MIMO identification method for wave energy applications in Faedo et al. (2019), particularly to approximate the response of an array of WECs, i.e. a “farm” of multiple 1-DoF devices. This strategy is based on the underlying theoretical concepts developed in Faedo et al. (2018b), and it allows for the computation of a model that exactly matches the frequency response of the target MIMO system at a set of user-selected frequencies  $\mathcal{F}$ , providing an efficient and accurate method to compute a state-space representation for the WEC dynamics. Additionally, a wise selection of the set  $\mathcal{F}$  within this moment-based approach helps to enforce the underlying (physical) properties of the WEC under analysis.

Motivated by these results, in this paper we present an adaptation of the MIMO identification framework developed in Faedo et al. (2019) to compute a parametric approximation of the radiation force subsystem of a multi-DoF device. We demonstrate that treating the approximation of radiation forces with our MIMO moment-based strategy (instead of the usual “multi-SISO” approach) provides a highly accurate low-dimensional system, hence offering a reliable parametric model while also reducing the computational effort required for time-domain simulations and control/estimation calculations. Moreover, we show that we can guarantee physical properties of radiation forces in the approximating model, such as bounded-input bounded-output (BIBO) stability.

The remainder of this paper is organised as follows. The section titled “Moments for MIMO Systems” recalls the theory behind moment-matching for MIMO systems. The section titled

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**KEY WORDS:** Radiation forces, parametric form, model order reduction, frequency-domain identification, moment-matching.