

An Explicit/Implicit Lie-group Scheme for Solving Problems of Nonlinear Sloshing Behaviors

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This paper presents an equal-norm multiple-scale Trefftz method (MSTM) associated with the group-preserving schemes (GPS) to tackle some difficulties in nonlinear sloshing behaviors. The MSTM combined with the vector regularization method is first adopted to eliminate the higher-order numerical oscillation phenomena and noisy dissipation in the boundary value problem. Then, the weighting factors of initial and boundary value problems are introduced into the linear system to prevent the elevation from vanishing without iterative computational controlled volume. More important, the explicit scheme, based on the $GL(n, R)$, and the implicit scheme can be combined to reduce iteration number and increase computational efficiency. A comparison of the results shows that the proposed approach is better than previously reported methods.

INTRODUCTION

Sloshing of liquid in tanks has received considerable attention from many researchers in related engineering fields. The problem arises because excessive sloshing of the confined liquid can strongly damage the structure or the loads induced by sloshing, which may seriously modify the dynamics of the vehicle that supports the tanks—for example, fuel sloshing in liquid propellant launch vehicles (Lu et al., 2015), oil oscillations in large storage tanks as a result of long-period strong ground motions (Hashimoto et al., 2017), and sloshing in nuclear fuel pools owing to earthquakes (Eswaran and Reddy, 2016). Besides, sloshing effects in the ballast tanks of a ship may cause it to experience large rolling moments and eventually capsize because of loss of dynamic stability (Krata, 2013; Sanapala et al., 2018). Also, if the forcing frequency coincides with the natural sloshing frequency, the high dynamic pressures, by reason of resonance, may damage the tank walls. Thus, accurate prediction of sloshing behaviors in tanks driven by external forces is very critical for successful structural design and reducing impacts on vehicle maneuvering.

The early work on nonlinear sloshing is based on asymptotic expansion techniques. For example, Penney et al. (1952) carried out a successive approximation approach where the potential function is expressed as a Fourier series with coefficients of functions of time. This method was applied by Skalak and Yarymovich (1962) and Dodge et al. (1965). Later, Chu (1968) used the characteristic functions to determine the subharmonic response to an oscillatory axial excitation. Bauer and Eidel (1988) expanded the nonlinear kinematic and dynamic free-surface conditions into Taylor series about the undisturbed free-surface level. In recent years, Lukovsky et al. (2012) used the Lukovsky–Miles variational method and the Narimanov–Moiseev asymptotics to deduce a nonlinear modal, which is applied to describe the resonant liquid sloshing in an upright cylindrical tank. For a long time, researchers have been driven to explore the behavior of

liquid motion in a forced and excited tank and seek for remedies of passive/active control of liquid sloshing by using physical mathematics, numerical simulation, and experiments (Bauer and Eidel, 1988; Celebi and Akyildiz, 2002; Rai et al., 2017). For physical mathematics, both linear and nonlinear mathematical models were proposed to provide the fundamental theory for sloshing analysis (Chen et al., 1996; Akyildiz, 2012; Viola et al., 2018). As for the numerical simulation, numerical methods for sloshing analysis can be roughly classified into the finite element method (FEM) (Okamoto and Kawahara, 1990; Mitra et al., 2008; Kolukula and Chellapandi, 2013; Nicolsen et al., 2017), the boundary element method (BEM) (Chen et al., 2007; Zhao et al., 2018; Chen et al., 2017), and the finite difference method (FDM) (Chen and Chiang, 1999; Kim, 2001; Zhang et al., 2016). The FEM is more versatile and can be applied to most engineering problems compared with the FDM and BEM. The BEM has some specific advantages over the FEM, such as reduction in computational dimension and ease in handling of data. All these methods, however, still have some drawbacks, such as element distortion, singularity of the fundamental solutions, costly computation, etc. Therefore, some meshless numerical methods have recently been proposed for constructing approximate solutions without suffering these defects, including the smooth particle hydrodynamics (SPH) method (Vaughan et al., 2008; Zhang et al., 2018), the meshless local Petrov–Galerkin method (Ma, 2005; Divya and Sriram, 2017), and the method of fundamental solutions (MFS) (Kołodziej and Mierzwiczak, 2008; Grabski and Kołodziej, 2018). For numerical errors and instability, most previous studies have reported that the nonlinear term is the main reason for free surface motion, and special techniques have been developed to track it. In common numerical approaches, there are three ways of describing the fluid motion. These approaches include the Eulerian description (Hirt and Nichols, 1981; González et al., 2017), the Lagrangian description (Okamoto and Kawahara, 1990; Battaglia et al., 2018), and the arbitrary Lagrangian–Eulerian method (Hirt et al., 1974; Yong and Baozeng, 2017).

To obtain an accurate solution of the linear equations under noisy effect, some literatures have reported the remedies. For example, Li and Huang (2008) and Li et al. (2013) studied the effective condition number and the stability analysis for the collocation Trefftz method (CTM) and the MFS. Ramachandran (2002) applied the singular value decomposition (SVD) on the MFS to

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