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A Regression Analysis Result for Water Waves on Irrotational Flow over a Horizontal Bed

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For the purposes of engineering applications in offshore structure design, an analytical approximation is presented. The solution is based on a Fourier series expansion. An approximation is derived with a geometric series, which is simple and closer to a Jacobi elliptic function in shallow water or the trochoid in deep water. By applying the approximation to the two boundary conditions on the free surface, the total error is obtained, which gives a functional to determine the related constants by regression analysis. As a regression analysis, the least squares method is considered. For the regression analysis, all field quantities are represented in dimensionless forms. The constants are presented with closed forms. The approximation is simple and valid for all waves including those in the solitary wave limit and in the deepwater wave limit.

INTRODUCTION

Since the early development of theory describing waves using the perturbation method of Stokes (1847), many nonlinear solutions have been obtained, both analytically and numerically. However, Airy wave theory is used in an even broader range than its validity range because of its simplicity. For shallow water waves, an interpolation between the solitary wave theory and Airy wave theory is used (ABS, 2007). Thus, a simple and accurate approximation is necessary for engineering application.

Stokes (1880) calculated waves in infinite/finite depths. De (1955) and Skjelbreia and Hendrickson (1960) obtained the fifth-order solutions for waves in finite depth. Schwartz (1974) and Cokelet (1977) obtained solutions of higherorder expansions. The Stokes theory and subsequent extensions of the theory are not generally applicable to shallow water.

Expansions are also made in series in terms of the shallowness giving rise to cnoidal wave solutions (Korteweg and de Vries, 1895). These are in terms of the Jacobi elliptic function *cn*, which is why they are coined cnoidal waves. They are used to describe waves of fairly long wavelength as compared to the water depth.

Parallel to the Stokes theory extension, series attempts have been made to develop a method that is equally valid for deep and shallow water. Based on the use of truncated Fourier expansions for field quantities, Chappelear (1961) and Dean (1965) developed a "Fourier approximation method." By choosing the expansions to satisfy the governing equation and bottom boundary condition, they reduced the problem to solving a set of nonlinear equations for each of the Fourier coefficients and, subsequently, the wave properties, such as surface elevation and wave speed. These include the limitation of the attainable accuracy introduced by the use of Simpson's rule integration in the solution process. Chaplin (1980) applied the Gram-Schmidt orthogonal process as an alternative to Dean's method and obtained improved results. However, since the stream function expansions contain hyperbolic functions, neither of the above stream function approaches can be applied for waves in deep water.

Following the rapid development in computing facility, considerable research efforts on the development of progressive wave theory have been made using numerical approaches. Rienecker and Fenton (1981) directly solved the coefficients by Newton's method, which made the solution process much simpler. Their results agreed well with the numerical results for arbitrary uniform water depth of Cokelet (1977) and with those in shallow water of Vanden-Broeck and Schwartz (1979), and the results for fluid velocities agreed well with the experimental data of Le Méhauté et al. (1968). This method was further simplified by Fenton (1988). The major simplification is that all the partial derivatives necessary are obtained numerically. In application of the method to waves that are high, in common with other versions of the Fourier approximation method, it was found that it is sometimes necessary to solve a sequence of lower waves, extrapolating forward in height steps until the desired height is reached. For very long waves, these methods can occasionally converge to the wrong solution. While this problem can be avoided by using a sequence of height steps (Fenton, 1990) it is simply solved by introducing an accurate approximation to the wave profile in this study.

The objective of this study is to provide an approximation for engineering applications and to provide the first step solution in Fenton's method. For infinite depth, the two problems were solved by Shin (2016) via a method that is simpler than and has less error than the Stokes theory. By introducing an approximation to the wave profile, the method is extended for finite depth in this study. The approximation is derived from the geometric series. It is simple and close to the Jacobi elliptic function in shallow water or the trochoid in deep water.

DIMENSIONLESS COORDINATE SYSTEM

Choosing spatial coordinates (x, y) so that the origin is located at a point on the still water line (SWL), the horizontal x-axis is in the direction of wave propagation and the y-axis points vertically upward, with t being time, the fluid domain is bounded by the seabed $y = -h$ and by a free surface $y = \eta(t, x)$. The problem is a

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KEY WORDS: Fourier series approximation, geometric series, wave breaking limit, regression analysis, progressing water wave, Fenton's method.