Numerical Evaluation of Double and Single Integral Versions of the Green Function for Wave Resistance Determination

Mirjam Fürth 1), Mingyi Tan 2), Zhi-Min Chen 2), Makoto Arai 1)

1) Faculty of Engineering, Yokohama National University
   Yokohama, Kanagawa, Japan

2) Faculty of Engineering and the Environment,
   University of Southampton
   Southampton, Hampshire, United Kingdom

ABSTRACT

Rayleigh damping is introduced into a potential flow based method in order to improve wave resistance determination. The problem is modelled using Kelvin sources with a translating speed. A dissipative 3D Green function is applied. The inclusion of Rayleigh damping removes the singularity associated with conventional Green functions. However, in this study it is shown that this benefit is purely mathematical and that the numerical complexity associated with the integration of a double integral is still best overcome by separating the double integral into two single integrals. The wave resistance for submerged ellipsoid is determined using a panel method.

KEY WORDS: Green function; Wave Resistance; Dissipative potential flow; Rayleigh Damping; Panel method; Single Integrals; Numerical Integration.

INTRODUCTION

Steady ship motion in calm water is a classical problem in ship hydrodynamics. Potential flow modelling is a common method to predict the wave resistance of ships. Potential flow based models often have a shorter computational time than models based on Reynolds-Averaged Navier-Stokes (RANS) equations. One benefit of potential flow is that the problem is reduced to the size of the wetted surface, whereas RANS solvers require the discretisation of the whole fluid domain. It is this that gives potential flow its major speed advantage. Because all ship properties are intertwined, it is not beneficial to dwell too much on one parameter. It is therefore used for a wide range of industry applications during early phases of ship design (Wilson et al, 2010).

Dissipation is an important factor in wave propagation. Waves decay with time and distance. In this study, the application of a dissipative three-dimensional Green function which includes Rayleigh damping is evaluated. The hypothesis is that by including more of the physical phenomena such as wave’s decaying the model will give a better estimation of wave making resistance. Previous evaluations using thin ship theory has shown promising results (Fürth et al, 2013).

Kelvin sources are used to model the presence of the hull. The influence of the Kelvin sources is usually expressed by a Green function. The Green function in its conventional form is highly singular, which calls for a single integral approach for evaluation.

THEORETICAL BACKGROUND

There are five different formulations of the Kelvin source based Green function:

1. The most well-known is the original expression by Michell (1898).
2. The most widely used expression was derived by Havelock (1928) and expanded in (1932), inspired by the work of Lamb (1926). The Havelock expression was modified by, among others, Lunde (1951).
3. Peters (1949) derived a Green function. A main benefit to the Peters formulation is the non-variable limits in the integrals.
4. The Bessho (1964) formulation is sparsely quoted in the literature. The details of the derivation were not fully explained until the 1980s, when Ursell (1984) proved that the order of integration could be changed, which had previously been disputed in the Bessho formulation.
5. The Demanche (1981) formulation is not widely used, possibly because it lacks practical advantages compared to the more well-known expressions.

A detailed comparison between expressions 1-3 was provided by Noblesse (1981). Most commonly, the Green function is approximated by polynomials as performed by Newman (1985, 1987a, 1987b). Newman (1985) introduced the solution for a simplified case where both the source and field point are located on the centre plane. The polynomials for the integral component that describe the local, non-radiation symmetric portion of the steady translating source potential are described in Newman(1987a). The far field or wave like disturbance is described in Newman (1987b).