Simulations of short-crested harbour waves with Variational Boussinesq Modelling

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ABSTRACT
Waves propagating from the deep ocean to the coast show large changes in wave height, wave length and direction. The challenge to simulate the essential wave characteristics is in particular to model the speed and nonlinear interaction correctly. All these physical phenomena are present, but hidden, in the Euler equations, a set of partial differential equations which are Newton’s momentum equation for incompressible fluid. Numerical simulation of the full 3D equations are for present hardware still too demanding. For that reason an equivalent but completely different formulation can be used: a formulation in surface variables only (dimension reduction to 2D), given by a dynamic variational principle leading to a Hamiltonian system. The challenge is then to approximate the kinetic energy; we do this by exploiting a minimization property of this functional, leading to the so-called Variational Boussinesq Model (VBM). In that way we obtain tailor made accurate results with consistent a Finite Element implementation. The VBM software can be used for many important applications in coastal engineering. As one illustration the simulation of short-crested waves entering a harbour will be shown; experimental data from Deltares are available to quantify the simulations. Accurate wave simulations can play an important role in the design of the lay-out of harbours.

KEY WORDS: Short-crested waves; harbour; Variational-Boussinesq Model; wave disturbance.

NOMENCLATURE

\(H\) Hamiltonian or total energy
\(K\) Kinetic energy
\(P\) Potential energy
\(\eta(x,t)\) Surface elevation(m)
\(\Phi(x, z,t)\) 3D fluid potential
\(\phi(x,t)\) 2D fluid potential at free surface
\(h(x)\) Water depth(m)
\(g\) Gravitational acceleration (ms-2)
\(V_g\) Group velocity (ms-1)

INTRODUCTION
An accurate prediction of wave characteristics is an important as well as a challenging task to be completed in planning, designing, operating and improving (existing) coastal structures, such as a harbour. The wave disturbance near and in the inner harbour is the most important factor that has to be investigated to support the best possible design of the harbour configuration.

Young (1999) categorized two general classes of wave models for calculating wave characteristics; phase averaged and phase resolved wave models. Wave characteristics near a harbour (where there are no complicated structures and diffraction is not very important) can be predicted reasonably well by using phase-averaged models. Then waves are calculated in an ‘average sense’, i.e. only the energy spectra of the wave is modeled, while the phases of the wave are assumed to be uniformly distributed. If the wave properties vary slowly in the order of a few wavelengths (when diffraction is not very important), the phase averaged model can be much more efficient than the phase resolved model. On the other hand, the phase resolved models calculate the complete wave dynamics: the amplitude and phase of the wave. Then diffraction can be calculated accurately, although it depends on which phase resolved models are being used. The phase resolved models require more computation efforts than the phase averaged models, hence the area of application of these models is smaller than the phase averaged models.

Phase resolved models are quite popular among coastal engineers to calculate wave penetration in a harbour. The mild-slope equation (Berkhoff, 1972; Dingemans, 1997) and Boussinesq-type of equations (Peregrine, 1967; Madsen and Sørensen, 1992; Nwogu, 1993) are widely used and more preferred because of their ability to calculate accurately the diffraction, shoaling, refractions and nonlinearity, especially for calculating specific processes such as harbour oscillations (Miles 1974; Mei 1983). Each of these phase resolved models have their own limitations. The mild-slope equation can calculate rather efficiently regular (monochromatic) waves, but it cannot deal with irregular (broad-band) waves such as wind driven waves. On the other hand, Boussinesq-type of models are able to deal with irregular waves over varying bathymetry and allowing nonlinearity, but these models are limited by their accuracy in

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