Lagrangian Experiment and Solution for Progressive Gravity Waves on a Sloping Bottom

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ABSTRACT

A third-order asymptotic solution in Lagrangian description for nonlinear water wave propagating over a sloping bottom is derived. The particle trajectories are obtained as a function of the nonlinear ordering parameter ε and the bottom slope α to the third order of perturbation. This solution enables the description of wave shoaling in the direction of wave propagation from deep to shallow water, as well as the successive deformation of wave profiles and water particle trajectories prior to breaking. A series of experiment are conducted to investigate the particle trajectories of nonlinear water wave propagating over a sloping bottom. It is shown that the present third-order asymptotic solution agrees very well with the experiments.

KEY WORDS: Lagrangian; sloping bottom; particle trajectory; wave breaking.

INTRODUCTION

The motion of a fluid particle within a propagating surface wave may be described by either observing the fluid velocity at a fixed position or the trajectory of a particle that is carried along with the flow. These alternative descriptions are called Eulerian and Lagrangian approach, respectively. For an incompressible fluid, the Eulerian approach is clearly preferred, because of the corresponding continuity equation is linear. It is also well known that the Eulerian description for a free surface can always be expressed in Taylor series at a fixed water level, which implicitly assumes that the surface profile is a differentiable single-valued function. In the Lagrangian approach, however, the surface elevation is specified through the position of a surface particle, i.e., a particle whose vertical parameter is equal to zero. Unlike an Eulerian surface, which is given as an implicit function, a Lagrangian form is expressed through a parametric representation of particle motion. Hence, the Lagrangian description is more appropriate for limiting the free surface motion, whereas this unique feature cannot be represented by the classical Eulerian solutions (Biesel, 1952; Naciri and Mei, 1993; Chen et al., 2005, 2006 and Buldakov et al., 2006).

The first water wave theory in Lagrangian coordinates was obtained by Gerstner (1802) who assumed the flow possesses finite vorticity. Miche (1944) proposed a perturbation method for Lagrangian solution to a second order for a gravity wave motion. Pierson (1962) also applied perturbation expansion to water wave problems with Lagrangian formulae and obtained a first-order Lagrangian solution. Buldakov et al. (2006) developed a Lagrangian asymptotic formulation up to a fifth-order for nonlinear water waves in deep water. However, all the theories mentioned above are rotational and are limited to the condition of uniform water depth. To date, only a limited few analytic solutions are derived for wave transformation on a planar beach in Lagrangian coordinates. Among them, Sanderson (1985) obtained a second-order solution in a uniformly stratified fluid with a small bottom slope in a Lagrangian system. Constantin (2001) considered the first-order Lagrangian solution for edge wave on a sloping beach. Chen and Hwang (2000) derived a Lagrangian solution in terms of beach slope α to the second order for a progressive wave propagating over a gentle plane slope, while Kapinski (2006) studied the run-up of a long wave over a uniform sloping bottom in Lagrangian description.

The purpose of this paper is to develop a nonlinear solution for surface waves propagating over a sloping bottom in a Lagrangian description and to compare the theory with a series of experiments. In order to examine the effect of a sloping bottom and wave steepness on surface waves, a perturbation expansion is used to derive an expression for the particle trajectories in terms of wave steepness ε and the bottom slope α to the third power. The asymptotic solutions for physical quantities related to the wave motion are then obtained up to the third order. Finally, to validate the accuracy of the analytical results, a series of laboratory experiments are performed. The Lagrangian properties of particle trajectories are shown to agree with the experimental data very well.

FORMULATION OF THE PROBLEM

Consider a two-dimensional monochromatic wave propagating on uniform slope without refraction as shown in Fig. 1. The negative x-axis is outward to the sea from the still water level (SWL) shoreline, while the positive y-axis taken vertically upward from the SWL, and the sea bottom is at y = −d = αx, in which α denotes the bottom slope.

The fluid motion in the Lagrangian representation is described by tracing an individual fluid particle. For two-dimensional flow, a fluid particle is distinguished by the horizontal and vertical parameters (x0, y0) known as Lagrangian labels. Then fluid motion is described by a set of trajectories x(x0, y0, t) and y(x0, y0, t), where x and y are in Cartesian coordinates. The dependent variables x and y indicate the position...