

## Wave Drift Forces Affected by Low-Frequency Oscillations

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### ABSTRACT

In the present work, the problem of interaction between the low-frequency surge motion of a floating body and ambient wave fields is considered to investigate the effects of the slow oscillations on the nonlinear wave loads. It is assumed that the frequency of the slow surge motion is much smaller than the incident wave frequency. A body-fixed frame following the low-frequency surge motion is adopted to describe the problem. Two small parameters, i.e. the incident wave amplitude and the low frequency, are used in the perturbation expansion to simplify the problem. So obtained boundary value problems for each order of potentials are solved by means of the hybrid method. The fluid domain is divided into two regions by a virtual circular cylinder around the body. The potential in the inner region is expressed by an integral over the boundary surface with a Rankin source as its Green function while it is expanded into a series of eigenfunctions in the outer region. As an extension of previous work, differential operators are suggested to solve the higher order potentials in the outer region efficiently.

**KEY WORDS:** Nonlinear wave forces, Low-frequency drift motion, Secularity, Higher-order, Boundary Element Method, Eigen function expansion,

### INTRODUCTION

When ocean vehicles work in waves, it is usually required to operate at an accurate position by means of mooring system or Dynamic Positioning System (DPS). Ocean waves might be considered of a collection of an infinite number of sinusoidal waves as a first order approximation:

$$\zeta(\mathbf{x}, t) = \sum_{i=1}^{\infty} \text{Re}[\zeta_i(\mathbf{x})e^{i\omega_i t}] \quad (1)$$

When the non-linearity is taken into account, the second order ocean waves can be shown as follows:

$$\zeta^{(2)}(\mathbf{x}, t) = \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} \text{Re}[\zeta_{ij}^{(2-)}(\mathbf{x})e^{i(\omega_i - \omega_j)t} + \zeta_{ij}^{(2+)}(\mathbf{x})e^{i(\omega_i + \omega_j)t}] \quad (2)$$

The wave components with the double frequency or the sum frequency are not so important from the view point of accurate positioning.

However, ocean vehicles might oscillate in the horizontal plane at a low frequency as a result of excitation by the nonlinear wave components with the difference frequency in (2). These nonlinear exiting forces are quadratic in wave height and recognized as the varying wave drift forces. They may be quantitatively small but the floating body could be drifted into a large excursion in the horizontal plane when the resonance occurs. Pinkstar (1974) reported their research on varying wave drift force and its spectrum was calculated for a platform in ocean waves.

When the platforms oscillate in the horizontal plane at a low frequency, the force, which is proportion to the square of wave height, has an influence to the body motion. One is proportion to the velocity of the low frequency motion while the other one is proportion to the acceleration. They are called wave drift damping and wave drift added mass respectively.

Conventional damping can be ignored since it is sure that the wave energy calculated by slow drift motion is small. So the wave drift damping plays an important role.

Wave drift added mass on the other hand, is important because added mass will shift the resonance frequency of the system. It can be observed that the spectrum of the wave drift force has steep gradient with respect to frequency, it will change significantly even as the resonance frequency of the system shifts only a little bit. So even if wave drift added mass is a second order force, precise evaluation is necessary.

To investigate the wave-drift added mass explicitly, the interaction problem between the low frequency surge motion of the body and the ambient wave fields is considered based on the potential theory in the present study. Consistent perturbation expansion with two small parameters i.e. wave amplitude and low frequency ( $\sigma$ ) is performed in a body-fixed coordinate system. The present approach was similar to the one suggested by Newman (1993) in the calculation of the wave-drift damping. Two time scales are adopted. One is associated with the body motion at the frequency  $\sigma$  and the other corresponding to the incident wave frequency  $\omega$ .

To solve the higher order potentials, Nihei et. al. (2005, 2006) has been developed the program code for lower order potentials by means of hybrid method which is a combination of boundary element method and eigen function expansion. A new analytical approach for higher order potential was derived in 2006. The potential, which is linear in wave amplitude but affected by the acceleration of low-frequency oscillation, is solved in the present work. Calculations of wave drift added mass acting on various floating bodies are carried out.