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A Co-Rotational 3D Beam Element for the Non-Linear Analysis of Offshore Structures

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ABSTRACT

In this paper, a co-rotational 3D beam finite element formulation is presented for the geometrically non-linear response analysis of structures assuming that displacements are large and rotations are small within an iteration process. Large rotations are obtained by successive small rotations during the total number of iterations of an equilibrium state. A special iteration algorithm is explained for eccentrically connected members by using a rigid arm methodology. Attention is also paid to flexible member connections so that introducing master and slave degrees of freedom are avoided. An incremental iterative solution algorithm is applied to find equilibrium states. A couple of examples are presented to demonstrate the performance of the element.

INTRODUCTION

In the practical design of structures, a linear analysis method is used for the response calculation. It is correct only if a linear relationship exists between the input and output of the response. This condition is satisfied under certain assumptions such as the structural material is linear elastic, deformations are small, boundary conditions are linear, local deformations at member connections are negligible, etc..., which are far from the reality of most offshore structural systems. In the offshore structural analysis, an important non-linearity occurs in the wave loading. This does not produce any difficulty if a deterministic analysis is carried out. Apart from the wave loading, some important non-linearities arise from the material origin, e.g. yielding and strain hardening, as well as from large deformations due to structural flexibilities. The material non-linearity (physical non-linearity) affects the stiffness of the structure as it occurs in the analysis of concrete structures and also in the elasto-plastic and limit state analyses of steel structures. The non-linearity due to large deformations (geometric non-linearity) is a considerable phenomenon which causes structural instabilities under a compressive loading state so that it receives a great attention in practice to find the buckling load as well as to investigate the post-buckling behaviour. In general, non-linear problems of solids and structures have been well formulated and the solution algorithms have been developed, see e.g. Crisfield (1991). When dealing with the geometrical non-linearity of a 3D beam an extra difficulty arises from large rotations in which case the vector representation of rotations are not valid any more. This leads to a path depending response. An excursion into this problem and solution methods are presented in Argyris (1982). These methods are subsequently adopted by, e.g., Simo and Vu-Quoc (1986), Crisfield (1990), Ibrahimbegovic (1995), Pacoste and Ericsson (1997), in the development of a 3D non-linear beam element. Different formulation methods for a non-linear beam element are available in the literature as depending on the assumption of a reference local co-ordinate system of the beam in which deformations are primarily defined. The total Lagrangian (TL) formulation is based on the original local-co-ordinate system of the beam (undeformed under geometric nonlinearities) and the updated Lagrangian (UL) formulation is based on a co-ordinate system which is defined in the previous equilibrium state, see e.g., Bathe and Bolourchi (1979), Mattiasson et al. (1985) and Felippa et al. (1994). The co-rotational formulation (CR) is based on a reference local co-ordinate system of the beam which is defined in the current deformation state so that it continuously rotates and translates with the beam element. In this formulation, the rigid body displacements and rotations are eliminated and only the local deformations are used in the calculation of the strain energy from which the internal forces and the tangent stiffness matrix are obtained. A co-rotational 3D beam elements was first introduced by Oran (1973) and then it has been developed further by numerous research workers among which contributions of, e.g., Crisfield (1990) and Pacoste and Ericsson (1997), should be addressed. It was reported by Felippa et al. (1994) that the TL formulation has some shortcomings, especially for elements with rotational degrees of freedom, while the CR formulation is more physical and easier to work. Solution of the non-linear finite element equation is carried out by using an incremental iterative procedure to obtain an equilibrium state. The load increment is the straightforward procedure, but its application is limited to a simple load-deformation path and it fails at extreme points. This