Proceedings of the Eighth (1998) International Offshore and Polar Engineering Conference
Montréal, Canada, May 24-29, 1998

ISBN 1-880653-34-6 (Set); ISBN 1-880653-37-0 (Vol. III)

A Finite Element Model of Gravity Waves with Free Surface

Bing Chen, Yucheng Li and Guozhang Lai
Dalian University of Technology
Dalian, China

ABSTRACT

A numerical wave tank is well established based on extended pseudo-concentration method and Sommerfeld absorbing boundary conditions. The efficiency of numerical wave tank is tested by comparison between numerical examples and physical model test, the results is reasonable.

INTRODUCTION

The numerical simulation of wave motion with free surface is of significant interest in practical engineering. For example, in estimation of the fluid load on marine structure near the still water surface or in shallow water zone. The most important and difficult problem in free surface flow is to determine the position of the free surface and applying boundary conditions on it. To solve this problem, many approaches have been developed for decades.

Excepted moving mesh technique in Lagrangian framework, there are many Eularien methods had been presented because of the advantage that arbitrary deformation of fluid can be simulated theoretically. The MAC (marker and cell) method was present in 1965 by Frencis H. Harlow and J. Eddie Welch (1965). Since then varied methods extended from MAC method have been developed including the well known VOF (volume of fluid) method (Hirt & Nichols, 1981). VOF method is successful in solving many problem. Unfortunately, both MAC and VOF method are based on Finite Difference Method, and they have trouble in dealing with complex boundary. To take advantage of the ability of dealing with complex boundary of Finite Element Method, Erik Thompson (1986) presented a FEM model of free surface flow named pseudo-concentration method. The approach assigns a pseudo-concentration throughout the mesh in such a manner that its value indicates the presence or absence of real fluids. In regions where the real fluid is present, the appropriate physical parameter such as effective viscosity and density is assigned. In those regions of mesh where the value of the concentration indicates that the real fluid has not yet present, an artificially low value of viscosity and density is used so as not to affect the flow of real fluid. So the considered domain of flow is divided into two phase: realfluid phase and artificial fluid phase. The governing equation of the pseudo-concentration 'F' is

$$\frac{\partial F}{\partial t}+u_i\frac{\partial F}{\partial x_i}=0 \quad (1)$$

But the effectiveness of the pseudo-concentration method is restricted by the instability caused by the use of artificial physical parameters for void section of the domain. An additional problem in pseudo-concentration method is the numerical oscillating in solving the pure advection Equation (1).

To overcome the disadvantages of traditional pseudo-concentration method, J. Ptera and V. Naschi (1996) extended the pseudo-concentration method to a special Lagrangian framework along the trajectories of the fluid particles. The novel aspect of the extended pseudo-concentration method is that do not treat the flow domain as a two phase system and only parts of the flow field which are full of real fluid at any given time are included in the solution scheme. Thus there is no need to use artificial physical parameters in this method. The solution of the free surface function equation in this model need only updating the free surface function 'F' by the value on the previous position along the trajectories of the fluid particles. The momentum equation and continuity equation are solved in Eulerian framework and free surface function equation is solved in Lagrangian framework.

In this paper the extended pseudo-concentration method is applied on numerical simulation of wave tank. The efficiency of the model is verified by the comparison between the numerical results and measured data in laboratory.

GOVERNING EQUATIONS

The governing Equations are the N-S equations and the continuity equation of two dimensional incompressible viscous fluid. To apply LES turbulent model, the dimensionless governing equations are as follows:

$$\frac{\partial u_i}{\partial t}+u_j\frac{\partial u_i}{\partial x_j}=-\frac{\partial P}{\partial x_i}+\frac{1}{Re}\frac{\partial}{\partial x_j}\left(\frac{\partial u_i}{\partial x_j}\right)+g_i+\frac{\partial}{\partial x_j}\left(\tau_{ij}\right) \quad (2)$$

$$\frac{\partial u_i}{\partial x_i}=0 \quad (3)$$