

Numerical Simulations of Free Surface Flow with Boundary-Fitted Coordinate Systems

Fumihiko Yamada and Kiyoshi Takikawa
Kumamoto University
Kumamoto, Japan

ABSTRACT

A free surface flow is simulated numerically using a finite difference method with the boundary-fitted coordinate system. The Simplified Marker and Cell (SMAC) method is employed for solving the Navier-Stokes equations. The procedure is developed using the mixed Eulerian-Lagrangian approach for analysis of nonlinear free surface boundary conditions. Smoothing and relocation techniques were used to enhance accuracy and stability of the calculations. Numerical results of the sloshing motions and uni-nodal standing oscillation in a rectangular tank are shown. The accuracy of numerical solutions is verified through comparison with analytical solutions.

KEY WORDS : Free surface flow, Moving boundary problem, Boundary-fitted coordinate system, Mixed Eulerian-Lagrangian approach, Free surface boundary condition, Numerical grid generation

INTRODUCTION

The motion of a fluid with a free surface is an interesting phenomena in many fields of engineering. Typical examples of free surface flow are sloshing in a tank and ocean wave motion. Generally, free surface flow problems are difficult to analyze theoretically and experimentally due to the nonlinearity of the governing equations. Free surface flows are moving boundary problems where the free surface configuration continuously deforms with time. Consequently, in the numerical analysis of this phenomenon, the velocity fields must be solved giving consideration to the variation of the free surface profile.

To date, many numerical studies have been carried out, and they can

be classified roughly into two groups. The one group includes phenomena are treated as boundary value problems assuming the existence of a velocity potential. The Boundary Element Method (BEM; Longuet-Higgins and Cokelet, 1976; Nakayama, 1983) belong to this group. This method has difficulty taking into account the viscous fluid forces, because the velocity fields are supposed the irrotational and non-viscous flow.

The other group includes phenomena that are formulated as fluid motions by solving the Navier-Stokes equations severely. The Finite Difference Method (FDM; Harlow and Welch, 1965; Hirt and Nichols, 1981) is in this group, and large deformation problems of a free surface (e.g. wave breaking) can be applied to some extent (Miyata et al., 1985; Park, J. C. and Miyata, 1994; Takikawa et al. 1997). Analyses utilizing this method are based on the Eulerian approach, thus, numerical diffusions cause the location of the free surface to smear.

In this study, to decrease the numerical diffusions on the free surface, the Boundary-Fitted Coordinate (BFC) method based on the Navier-Stokes equations was adopted. In this method, the free surface is treated as a fixed boundary in mapping (transformed or computational) space, and the boundary conditions on the free surface are formulated exactly. Many conventional studies using the BFC method (Chiba and Kuwahara, 1989, Nagahama et al., 1992) assumed the free surface to be a single-valued function. Consequently, these studies cannot treat the over-turning jet of the plunging breaker where the free surface is a many-valued function.

Therefore, the BFC system with the Lagrangian approach used for the kinematic boundary condition on the free surface is used here. This permits over-turning jet to some extent. These systems include boundary-fitted coordinates and numerical grid generation. Generally, this procedure is called the mixed Eulerian-Lagrangian approach.