

## On the Fourier Series Decomposition of Directional Wave Spectra

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### Abstract

The estimation of directional wave spectra by means of normalized cross-spectra from a circular wave gage array is investigated. A Fourier series decomposition method is used. Problems and solutions connected with the extrapolation of a truncated directional Fourier series is studied. The truncation determines the directional resolution of the array. A relation between the truncation order and the relative size of the array is empirically found. Numerical examples on directional estimates, including various single-peak and double-peak spectra, illustrate this. The method is seen to work fine with large array. Use of Maximum Entropy extrapolation of the Fourier series enhances the resolution of a small array,

### INTRODUCTION

A detailed description of the ocean waves requires directional information as well as the commonly used scalar energy spectra. This has been a field of constant development during the last 20 - 30 years. Wave directionality can be quite significant to ocean engineering and naval architecture problems, and the use of directional spectra is growing. Several new multidirectional wave facilities are established around the world. Compared to the unidirectional case, the multidirectional wave problem is certainly more complex and challenging, for the wave modeller as well as for the end user. One of the challenges is to establish a consistent and robust way of determining and documenting the directional spectra. This can only be finally obtained by international communication and common efforts from different people and laboratories. One such example was the IAHR Working Group on Multidirectional Waves, whose work is summarized in Briggs (1997). During the work of this group, methods were compared against each other, and a list of standard symbols and definitions was proposed. Among the key issues and questions arising on the background of this and other works, is the balance between directional resolution and statistical variability. In this problem, the choice of methods and devices for directional measuring and analysis is one essential problem, while the assessment of natural and expected statistical scatter is another. The latter problem has been discussed in some detail in Stansberg (1998). In the present work, we shall look into

the directional resolution of a particular analysis method based on cross-spectra obtained from a wave gage array. There exist a number of different methods for such use, as shown in the review of Benoit et. al. (1997). Our study will focus on the Fourier Series decomposition method, which is a reasonably simple method to understand and interpret. The work is based on an earlier study in Stansberg & Ishida (1989). Here we shall in particular look into the problem of truncation and extrapolation of the directional Fourier Series. A particular method is proposed, partly based on Maximum Entropy Extrapolation. Numerical examples on use with idealized cross-spectra will be shown,

### DIRECTIONAL ESTIMATION BY FOURIER SERIES DECOMPOSITION

#### Directional spectra, coherence spectra and complex Fourier coefficients

The following is a summary with definitions and formulations, included as a background for the main part of our study. For a more general and comprehensive description of this problem we refer to previous works, such as e.g. the recent review in Benoit et. al. (1997).

We define the directional spectrum  $S(f,\theta)$  of a multidirectional wave field  $\eta(\mathbf{r},t)$  as:

$$S(f,\theta) = \langle |F(f,\theta)|^2 \rangle \, df \, d\theta \quad (1)$$

Here  $\eta(\mathbf{r},t)$  is the zero-mean wave elevation in time  $t$  and horizontal space  $\mathbf{r} = (x,y)$ , defined over all  $\mathbf{r}$  and  $t$ ; the brackets  $\langle \rangle$  means statistical averaging (expectation value);  $| |$  means modulus;  $f$  is the wave frequency (in Hz);  $\theta$  is the wave direction (in radians); and  $F(f,\theta)$  is defined by:

$$\eta(\mathbf{r},t) = \int_{-\infty}^{\infty} \int_{-\infty}^{2\pi} df \, d\theta \, F(f,\theta) \exp[j(2\pi ft - \mathbf{k}(f,\theta) \cdot \mathbf{r})] \quad (2)$$

Thus  $F(f,\theta) \, df \, d\theta$  can be interpreted as the complex amplitude of a plane harmonic wave component with frequency  $f$  and direction  $\theta$ . Here  $\mathbf{k}(f,\theta)$  is the corresponding wave vector with its modulus  $k$  given by the linear dispersion:  $(2\pi f)^2 = gk \tanh(kh)$ , with  $g$  = acceleration of gravity, and  $h$  = water depth. "j" is the imaginary unit. In principle, as indicated