

## Velocity Fields in Solitary Internal Waves

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### ABSTRACT

Internal waves propagating on the ocean thermocline pose a hazard for engineering operations in deep water (Osborne et al., 1978). The sheared currents associated with these waves can reach 1–2m/s and are present from the surface to the sea floor. From the viewpoint of the engineer, the most significant internal waves appear to travel as packets of solitary waves (or ‘solitons’), generated by tidal flow over uneven bathymetry (Maxworthy, 1980). Substantial work has already been completed on the kinematics of regular internal waves and a comparison has been made with non-linear regular wave theory (Martin & Easson, 1997). The current study extends this work to cover solitary waves and involves the generation of internal solitons in a laboratory channel which is 6m long and 0.4m wide. A paddle is moved vertically to generate a solitary wave, and wave gauges are used to monitor its passage along the channel. Differing layer depths and density stratifications are used in the experimental programme in order to cover a range of parameters. Particle Image Velocimetry (PIV) is used to obtain the velocities within a section of the wave. The velocity values are compared with those obtained using K-dV theory in order to assess the range of applicability of this theory and enable offshore engineers to better predict the forces to be expected under such conditions. Koop and Butler (1981) assessed the validity of various solitary wave theories against wave shape parameters, using a combination of water and Freon to create the stratification. This study extends their work by using miscible fluids (fresh water and salt solution) to create a diffuse pycnocline, employing continuously stratified K-dV theory to calculate the wave generation parameters.

**KEYWORDS:** Internal Waves, Solitons, Particle Image Velocimetry (PIV), Wave generation, stratified flow.

### 1.0 BACKGROUND

Internal waves are gravity waves which propagate in the interior of a stably stratified fluid. Such stratification exists in the ocean, in the atmosphere and in lakes. The stratification may take the form of a relatively abrupt increase in density with depth, such as in the ocean thermocline, or a more gradual increase over hundreds of metres in depth. Internal waves can propagate in a number of ways. These include short regular waves, cnoidal and solitary waves, wave rays and internal tides.

The main features of short thermocline waves are: 1) the stratification is confined to a relatively thin layer between deep homogeneous layers and 2) the wavelengths are short in comparison to the depths of the homogeneous layers. While it is certain that such waves play a significant role in the energetics of the thermocline they are not likely to be the main threat to offshore operations.

Korteweg and de Vries (1895) solved the problem of how long, finite amplitude surface waves could propagate without becoming unstable. Their theory applies where the wavelength is great compared to the water depth, producing waves with a  $cn^2$  surface profile, called cnoidal waves. As the wavelength increases to infinity, the wave height also increases and the  $cn^2$  shape turns into a  $sech^2$  solitary wave. Korteweg-de Vries (K-dV) waves are thus long waves with great stability. The theory is however only applicable to moderate wave heights, and the problem of what happens when wave heights become great has not been resolved.

Benjamin (1966) converted K-dV theory into an internal wave theory for any stratification. It has become very common to use the two-layer approximation, although the continuous stratification model, which is very appropriate to an ocean thermocline, is also easy to use.

Benjamin (1967) and Ono (1975) discovered a new class of long waves which apply only to stratified fluids. These are known as algebraic solitary waves or Benjamin-Ono (B-O) waves. B-O waves can exist in many modes where the wavelength is great compared to the thickness of a diffuse thermocline, even if the thermocline is not near the surface or sea floor.

The deep ocean is often continuously stratified even away from the thermocline. A linear or exponential increase in density with depth through the whole water depth can be a good approximation to the stratification. Under these conditions, internal waves are not confined to thermoclines but can propagate at angles to the horizontal throughout the water column. Their behaviour is well defined, however, since their angle of propagation is governed by the wave frequency and the local stratification. Thus, from an isolated disturbance, internal waves may propagate as rays (Holloway, 1997).

Surface (barotropic) tides can excite internal wave motions at the tidal frequency. These are known as internal tides or baroclinic tides, and their wavelengths can be several tens of kilometres. Internal tides can interact strongly with topography, generating other types of internal waves.