

Non-Gaussian Extremes in Numerically Generated Second-Order Random Waves on Deep Water

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Abstract

The statistics of second-order deep-water random wave trains is investigated through a large and systematic set of numerically generated records. In particular, the properties of extreme crest heights are considered. A previously described numerical synthesis procedure, based on second-order random wave theory, is applied. Ocean wave elevation records of 4.5 hours duration are generated for a variety of sea states including storm conditions. Each sea state is simulated with 48 independent realisations in order to assure robust statistics. Thus average (expected) values as well as sampling variabilities are worked out. Parameters of the study include skewness and kurtosis values, extreme crest heights as well as the asymmetry of individual extreme waves. Comparisons to available theory are made, verifying that the simulations work satisfactorily. For a 100-year sea state with $H_s=15\text{m}$ and $T_p=14\text{s}$ an increase of 15% is observed in the expected extreme crest height, relative to the linear case. The corresponding average extreme wave asymmetry factor is increased from 0.50 to 0.58. A considerable sampling variability, with a standard deviation corresponding to about 2m for the extreme crest in this sea state, is also observed.

INTRODUCTION

The nonlinear asymmetry of steep waves on deep water has been previously documented through several studies in the literature. Among other works, we mention here the full scale data analysis by Marthinsen and Winterstein (1992), and Vinje and Haver (1994). Laboratory works on the same matter also exist, such as e.g. analysis by Stansberg (1991), (1993), and Kriebel and Dawson (1993). In storm sea states, this asymmetry leads to extreme crest heights that can become significantly higher than those predicted by linear theory and Rayleigh distributed peaks. For a 100-year storm, an increase of at least 15% should be expected for the most extreme crest heights. Most of this asymmetry can be explained by second-order nonlinearities in the wave field. At least, this is seen from the statistical skewness values in the full scale records referred to above. When it comes to the corresponding extreme crest levels, a certain contribution from third- and higher order effects

should also be expected, but the second-order contribution is nevertheless the first step upwards, and normally the most important one, from the linear description. Knowledge about effects introduced by second-order corrections is therefore of great value in the modelling of random wave trains and their extremes. The purpose of the present paper is to investigate the statistics of these effects through a set of systematic numerical simulations of second-order random wave records, combined with comparisons to available theory to verify the simulations.

The theory of second-order random waves on deep water was described by Longuet-Higgins (1963). In our study this is a basic reference for the numerical simulations as well as for theory comparisons. We also make use of results from later works, such as the statistical modelling in Marthinsen and Winterstein (1992) and in Winterstein (1988), and the numerical modelling procedure in Stansberg (1993). In the following, a brief review of this background is first given. Results from the simulations are then presented. The results include statistics on parameters such as skewness and kurtosis, extreme crest height levels, and asymmetry of extreme wave heights. In this analysis, the statistical averages as well as the corresponding sampling variability of the simulations are considered.

SECOND-ORDER RANDOM WAVES: BASIC FORMULATION

The formulation used in this study has been presented earlier, see e.g. Stansberg (1993), (1994), but essential items are reviewed below for completeness. A similar approach is also used in Marthinsen and Winterstein (1992).

Let an arbitrary and finite, but long record of the stationary random (irregular) surface wave elevation $\eta(t)$ at a given location be described as a second-order signal composed as follows:

$$\eta(t) = \eta^{(1)}(t) + \eta^{(2)}(t) \quad (1)$$

where $\eta^{(1)}(t)$ is the first-order (linear) term and $\eta^{(2)}(t)$ is its corresponding second-order additional term. The first-order term can be written in terms of its complex Fourier transform $N(f)$ as: