

The Soil Rigidity Effect in the Touchdown Boundary-Layer of a Catenary Riser: Static Problem

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ABSTRACT

Recently, analytical solutions, of the boundary-layer type, were developed for the static and dynamic curvature problems in the touch-down point (TDP) region of a catenary riser (Aranha, Martins & Pesce, 1997; Pesce et al., 1997). Nevertheless, the analysis was restricted to the infinitely rigid soil assumption. The present paper enlarges somewhat the static boundary-layer solution, by considering the existence of a linearly elastic soil. A nondimensional soil rigidity parameter, $K = k\lambda^4/EJ = k\lambda^2/T_0 = kEJ/T_0^2$, is defined, where k is the rigidity per unit area, EJ the bending stiffness, T_0 the static tension at TDP and λ is the flexural-length parameter representing the TDP boundary-layer length scale. Nondimensional - hence generally valid - diagrams, having K as parameter, are presented, showing, $K \geq 10$, the *elastica*¹, the horizontal angle, the shear effort, and the curvature, as functions of the local nondimensional arclength parameter, s/λ .

Keywords: catenary riser, static problem, touch down point, soil effect.

INTRODUCTION

The precise evaluation of the curvature at TDP region is one of the toughest problems in catenary riser design. In formulating and solving such a problem by means of full nonlinear codes based on the Finite Element Method, the soil elasticity effect has often been addressed by the use of elastically supported contact elements. In general, as could be expected, it has been found that such an effect smoothes the sudden variation that appears in the shear force when the soil is considered to be infinitely rigid. However, as in any analysis supported solely on numerical methods, an overall picture is hard to achieve.

On the other hand, recognizing that for a free hanging catenary riser, the flexural rigidity effect is confined and dominant just inside small regions close to the touch-down point (TDP) (and also to the top end), a standard boundary-layer technique was applied in order to construct local consistent analytical solutions for the static and dynamic curvature problems (Aranha, Martins & Pesce, 1997; Pesce et al., 1997), that match smoothly the corresponding 'outer' ideal cable solution. Nevertheless, this previous analysis was restricted to the infinitely rigid soil assumption. In that particular but important asymptotic case, a flexural-length parameter, $\lambda = \sqrt{EJ/T_0}$, that measures the boundary-layer length scale, has been shown to play a significant role on the local solution, being properly interpreted as the distance between the actual and the ideal cable TDP. In the above formula EJ is the bending stiffness and T_0 is the tension at the TDP.

The present paper enlarges somewhat the static boundary-layer solution, by considering the existence of a linearly elastic soil. The constructed solution shows, as expected, a typical oscillatory behavior for the *elastica* on the supported part of the line, and how this behavior matches smoothly the catenary solution along the suspended part. Instead of being a measure for the position of the actual TDP, with respect to the ideal cable configuration, in the linearly elastic soil problem this flexural length parameter is shown to measure the displacement of the point of horizontal tangency. A nondimensional soil rigidity parameter, $K = k\lambda^4/EJ = k\lambda^2/T_0 = kEJ/T_0^2$, is defined, where k is the rigidity per unit area. Nondimensional (generally valid) diagrams, having K as parameter, are presented, showing, as functions of the nondimensional arclength parameter s/λ , ranging within the boundary-layer, the local solution for the *elastica*, the horizontal angle, the shear effort, and the curvature. Also, another nondimensional diagram is presented, showing the actual TDP position as a function of K .

¹ For conciseness sake, the term *elastica* is used herein, meaning the elastic line in the sense of Love, 1927, p. 2, that represents a bent rod, in which "the resistance to bending is a couple proportional to the curvature of the rod when bent", i.e., the classical constitutive relation, $M = EJ\chi$, applies, being M the bending moment and χ the curvature.