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A Finite Element Model of Gravity Waves with Free Surface

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ABSTRACT

A Finite Element Method (FEM) numerical model of wave tank is well established based on the extended pseudo-concentration method and Sommerfeld radiation boundary conditions. The efficiency of a numerical wave tank is tested by comparison between numerical examples and by a physical model test; the results are reasonable.

INTRODUCTION

The numerical simulation of wave motion with the free surface is of significant interest in practical engineering, for example, in estimating the fluid load on a marine structure near the still water surface or in a shallow water zone. The most important and difficult problem in free-surface flow is to determine the position of the free surface and apply boundary conditions to it. To solve this problem, many approaches have been developed for decades.

Excepted the moving mesh technique in a Lagrangian framework, many Eularian methods have been presented because of the advantage that arbitrary deformation of fluid can be simulated theoretically. The MAC (marker and cell) method was presented by Harlow and Welch (1965). Since then varied methods extended from the MAC method have been developed, including the well-known VOF (volume of fluid) method (Hirt and Nichols, 1981). The VOF method is successful in solving many problems. Unfortunately, both the MAC and VOF methods are based on the Finite Difference Method, and they have trouble dealing with a complex boundary. To take advantage of the ability of dealing with the complex boundary of the Finite Element Method, much work has been done in the last two decades, for example, Nakayama (1996) and Medale (1997). Thompson (1986) presented a FEM model of free-surface flow named the pseudo-concentration method. The approach assigns a pseudo-concentration throughout the mesh in such a manner that its value indicates the presence or absence of real fluids. In regions where the real fluid is present, the appropriate physical parameter such as effective viscosity and density is assigned. In those regions of mesh where the value of the concentration indicates that the real fluid is not yet present, an artificially low value of viscosity and density is used so as not to affect the flow of real fluid. So the considered domain of flow is divided into 2 phases: the real fluid phase and the artificial fluid phase. But the effectiveness of the pseudo-concentration method is restricted by the instability caused by the use of artificial physical parameters for the void section of the domain. An additional problem in pseudo-concentration method is the numerical oscillating in solving the pure convection equation of the pseudo-concentration function.

To overcome the disadvantages of the traditional pseudo-concentration method, Ptera and Nassehi (1996) extended the pseudo-concentration method to a special Lagrangian framework along the trajectories of the fluid particles. The novel aspect of the extended pseudo-concentration method is that it does not treat the flow domain as a 2-phase system and only those parts of the flow field which are full of real fluid at any given time are included in the solution scheme. Thus there is no need to use artificial physical parameters in this method. The solution of the free-surface function equation in this model needs only updating the pseudo-concentration function by the value on the previous position along the trajectories of the fluid particles. The momentum equation and continuity equation are solved in the Eulerian framework and the equation of pseudo-concentration function is solved in the Lagrangian framework.

In this paper the extended pseudo-concentration method is applied to the numerical simulation of a wave tank with a pipeline located near the bottom; both wave surface elevation and forces acting on the pipeline model are calculated. The numerical results are compared with the physical-model test results of Li, Kang et al. (1996).

GOVERNING EQUATIONS AND SOLVING METHOD

The governing equations are the N-S equations and the continuity equation of a 2-dimensional incompressible viscous fluid. To apply the LES turbulent model, the dimensionless governing equations are as follows:

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{\partial P}{\partial x_i} + \frac{1}{\mathrm{Re}} \frac{\partial}{\partial x_j}\left(\frac{\partial u_i}{\partial x_j}\right) + g_i + \frac{\partial}{\partial x_j}(\tag{1}$$

$$\frac{\partial u_i}{\partial x_i} = 0 \tag{2}$$

$$\text{where } \tau_{ij} = \nu_s \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \tag{3}$$

$$\nu_s = (c\Delta)^2 \left[\frac{\partial u_i}{\partial x_j}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)\right]^{\frac{1}{2}} \tag{4}$$

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KEY WORDS: Finite element model, numerical wave tank, extended pseudo-concentration method.