

## Fully Nonlinear Numerical Simulation of Self and Dual Wave-Wave Interaction

A.H. Clément\*

Laboratoire de Mécanique des Fluides - Division Hydrodynamique Navale,  
CNRS URA 1217, Ecole Centrale de Nantes, Nantes, France

L. Gil

Departamento de Engenharia Mecânica - F.C.T - U.N.L  
Quinta da Torre, Portugal

### ABSTRACT

This paper recaps and extends the results presented in Clément and Gil (1997). The interaction between two colliding monochromatic wavetrains in water of finite depth is investigated through numerical experiments. A two-dimensional numerical wave tank is used for this purpose. Each end of this flume is a piston acting as a wavemaker; it can be switched to be a wave-absorber when the wave packet emitted from the other end reaches it, preventing their re-reflection back in the basin. The main effect of the nonlinear interaction during the head-on collision consists in a phase lag of the wavetrain due to a decrease of the phase velocity. The influence of the frequency, the amplitude and the wave number of the 2 incident waves on the phase lag is investigated by numerical experiments using a fully nonlinear numerical wave tank (CANAL). The analysis of the self-interaction problem (i.e., when both waves have the same frequency and amplitude) presented in the above-mentioned ISOPE Conference paper is supplemented herein with the case of dual interaction (waves with different amplitude and frequency). Results established analytically by perturbation methods for infinite water depth are recovered here and numerically validated for finite water depth conditions.

### INTRODUCTION

The nonlinear interaction between two or more different water wavetrains has been widely studied using perturbation theories (Longuet-Higgins, 1962; Phillips, 1960; Su and Mirie, 1980), Fourier (Fenton-Rienecker, 1982) or spectral methods (Dommermuth-Yue, 1987), or numerical simulations (Grilli et al., 1989; Olmez-Milgram, 1995). The same problem was also investigated under the assumption of viscous flow by Gil and Gentaz (1997). Being exempt of assumption, except inviscid fluid and being limited only by (numerical) wave breaking, the *numerical wave tank* approach has proved to be very useful in investigating this kind of nonlinear free surface phenomenon. In the present work, it is used to investigate the effect of head-on collision of short nonlinear gravity waves in water of finite depth. They are *short* in the sense that their wavelength is not long compared to the water depth; the latter is then finite but not shallow.

In the case of 2 long (solitary) waves, the nonlinear interaction results in a time shift and the emission of a dispersive wavetrain in the soliton tail (Su and Mirie, 1980). When a long wave interacts with short dispersive waves, both of them experience a Doppler effect and a shift in phase (Clamond, 1994).

We shall see in this paper that for 2 short waves, the same effect is observed on the phase and that it may be calculated, in some finite waterdepth cases, by a formula proposed by Longuet-

Higgins and Phillips (1962) for infinite depth.

The problem is stated in 2 dimensions in the general framework of free surface potential flows. No assumptions other than inviscid fluid are made and the nonlinear initial boundary value problem is solved in the time-domain using a direct Mixed Euler-Lagrange BEM method (Clément and Mas, 1995; Clément, 1996b). Each end of the numerical wave tank acts either as a piston wavemaker generating monochromatic wavetrains modulated by an envelope function, or as a piston wave-absorber (Clément, 1996a).

### THE NUMERICAL WAVE TANK

In all the subsequent equations the physical variables are nondimensionalized with respect to the water depth  $h$  for the length variables, and to  $(h/g)^{1/2}$  for the time variable. The fluid density is set to unity.

The usual assumptions of free-surface potential flow theory are made: The fluid is incompressible and inviscid, surface tension is neglected, the atmospheric pressure above the free surface is constant and chosen as the pressure reference, the flow is irrotational. The fluid is at rest at  $t = 0$ .

Owing to the above assumptions, the fluid velocity  $\vec{V}$  derives from a scalar potential  $\Phi(M, t)$  function of time and space:

$$\vec{V}(x, y, t) = \vec{\nabla}\Phi(x, y, t) \quad (1)$$

The potential  $\Phi$  is easily shown to be the solution of the following nonlinear initial boundary value problem (BVP):

$$\Delta\Phi(x, y, t) = 0 \quad \text{in the fluid domain } \mathcal{D}(t) \quad (2)$$

$$\frac{\partial\Phi}{\partial y}(x, -1, t) = 0 \quad \text{on the sea bottom } \mathcal{B}(t) \quad (3)$$

\*ISOPE Member.

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KEY WORDS: Water waves, wave interaction, numerical simulation, numerical wave tank, potential flow.