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Propagation of Surface Waves of Finite Amplitude in a Basin with Floating Broken Ice

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ABSTRACT

A propagation of periodic traveling waves of finite amplitude over the plane bottom in homogeneous fluid covered by floating broken ice is considered. The dependence of the wave profile on the ice thickness and characteristics of an initial harmonic are analyzed. The ice effect on the Stokes' drift velocity and total mean mass transport is studied.

INTRODUCTION

Using the infinite small wave assumption, many theoretical studies have been conducted to investigate the propagation of surface gravity waves under floating ice fields (Peters, 1950; Weitz and Keller, 1950; Kheisin, 1967; Bukatov and Cherkessov, 1971; Bukatov, 1975; and Wadhams, 1986). These studies show that the influence of the ice field on the waves decreases with increasing wave period. The long-period waves of small amplitude travel under the ice without noticeable distortions.

Murty and Polavarapu (1979) noted certain inconsistencies between results of known theoretical investigations of the ice effect on long-period waves and real data. Therefore, further investigations with the aim of a more precise definition of the role of floating ice in the wave dynamics are advisable.

In this paper we consider the effect of floating broken ice on periodic traveling surface waves of finite amplitude in fluid of constant depth. The uniform approximate expansions up to values of third order for the fluid velocity potential and elevation of the fluid's surface are obtained by using the method of multiple scales. The dependence of amplitude and spatial profile of a nonlinear wave on the ice thickness and characteristics of an initial harmonic are studied. The expressions allowing definition of mean non-zero transport of the fluid particles are obtained. A quantitative estimate of the ice effect on Stokes' drift and total mean mass transport is given.

PROBLEM STATEMENT

Let us consider a basin unbounded in horizontal directions and of constant depth H. This basin is filled with inviscous incompressible fluid. Its surface is covered by floating broken ice. We study the influence of the ice on periodic traveling waves of finite amplitude, assuming the friction of floating floes to be negligible. We also assume that the floes are small in comparison with the wavelengths, and oscillations of the floes are nonseparating. Under mentioned assumptions, the floes' bend does not occur. In this connection, we take into account the gravity force as a single restoring force. Introduce the dimensionless variables $x = kx_1$, $z = kz_1$, $t = \sqrt{kg}t_1$ where k is the wavenumber. Then in the case of potential movement of the fluid, we have the following problem:

$$\Delta\varphi = 0,\ -\infty < x \le \infty,\ -H < z < \zeta \tag{1}$$

with boundary conditions at the surface ($z = \zeta$)

$$\zeta - \frac{\partial\varphi}{\partial t} + \kappa k\left(\frac{\partial^2\varphi}{\partial z\partial x}\frac{\partial\varphi}{\partial x} - \frac{\partial^2\varphi}{\partial t\partial z}\right) + \frac{1}{2}\left[\left(\frac{\partial\varphi}{\partial x}\right)^2 + \left(\frac{\partial\varphi}{\partial z}\right)^2\right] = 0 \tag{2}$$

and at the basin's bottom ($z = -H$)

$$\frac{\partial\varphi}{\partial z} = 0 \tag{3}$$

At the initial moment ($t = 0$):

$$\zeta = f(x),\ \frac{\partial\zeta}{\partial t} = 0 \tag{4}$$

Here $\kappa = h\rho_1/\rho$, h and ρ_1 are the thickness and density of the ice, ρ is the fluid density, g is the acceleration of gravity. The velocity potential φ and basin's surface elevation ζ are connected via the kinematic relation:

$$\frac{\partial\zeta}{\partial t} - \frac{\partial\zeta}{\partial x}\frac{\partial\varphi}{\partial x} + \frac{\partial\varphi}{\partial z} = 0 \tag{5}$$

A term with multiplier κ in the dynamic condition (Eq. 2) represents the inertia of the ice vertical displacements. And also the first term in brackets of this expression characterizes the nonlinearity of the vertical acceleration of the ice.

EQUATIONS DEFINING NONLINEAR APPROXIMATIONS

We find a solution to the problem defined by Eqs. 1~5 using the method of multiple scales (Nayfey, 1976). Such a technique allows us to obtain the uniformly converging expansions for ζ and φ. Let us introduce 2 new variables $T_1 = \varepsilon t$, $T_2 = \varepsilon^2 t$, which are

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