

## Heave Oscillations of a Submerged Vertical Cylinder in a Viscous Fluid

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### ABSTRACT

Nonlinear axisymmetric viscous and inviscid solutions corresponding to the heave oscillation of a submerged vertical cylinder are presented. The fully nonlinear free-surface flow problem is solved using the finite-difference method based on curvilinear coordinates. A primitive-variables based fractional-step formulation is used for the analysis of viscous flow, and a mixed Eulerian-Lagrangian formulation for that of inviscid flow. New results obtained for a range of amplitudes and frequencies of body oscillation and mean depths of body submergence demonstrate the effects of a free surface on the dynamics of body-generated vortices and on the vortical component of hydrodynamic force acting on the body. The results indicate the occurrence of negative added mass at intermediate frequency when the submergence of the cylinder is small. Force calculations reaffirm that the effect of viscosity on hydrodynamic forces is of significance at low and intermediate frequencies.

### INTRODUCTION

The study of near-surface wave-body interactions is of significance in many ocean engineering and naval architecture applications. The prediction of motion response of ocean vehicles to surface waves, determination of wave loads on offshore structures, design of buoy systems for oceanographic measurements are examples of such applications. From a scientific viewpoint, the study is of importance for understanding vorticity generation, vortex dynamics, and flow turbulence in the presence of a free surface. In contemporary engineering practice, a wave-body interaction problem is often tackled expeditiously using the Morison-equation approach. This kind of an approach is however of little value in problems involving, for example, slightly submerged bodies. These problems are highly nonlinear and the interaction between wave and body-generated vortical motions could be a significant factor affecting the hydrodynamic loading on the body. A rigorous analysis of nonlinear wave-body interactions in viscous fluids is necessary for an accurate estimation of fluid loads on bodies near the free surface. Such an analysis, as pursued in the present work, is expected to eventually phase out the use of ad hoc methods now prevalent in ocean engineering applications.

The specific purpose of the present work is to rigorously examine the importance of viscosity and free-surface nonlinearity in three-dimensional wave-body interactions encountered in ocean engineering applications. To achieve this objective, we have developed a robust and accurate finite-difference method based on curvilinear coordinates for the solution of the viscous free-surface flow problem. The present method is an extension of our earlier work developed for two-dimensional analysis (Ananthkrishnan, 1991; Ananthkrishnan and Yeung, 1994; Yeung and Ananthkrishnan, 1992). In order to quantify effects of viscosity, by comparison, we have also developed a finite-difference method to solve the counterpart inviscid-flow problem. In this paper, we describe these solution methods and present nonlinear axisymmetric viscous and inviscid solutions obtained for the heave oscillation of a submerged vertical cylinder.

These new results reveal some important effects of viscosity at low frequency of oscillation and thus, in a practical scenario, are of relevance to the dynamics of compliant structures such as tension-leg platforms, which tend to have low natural frequency. Also, as observed in earlier two-dimensional studies (Hodges and Webster, 1986; Ananthkrishnan and Yeung, 1992), the flow is highly nonlinear even at a very small amplitude of oscillation, when the body is slightly submerged. The presence of a free surface affects the evolution of body-generated vortices and thus the vortical-component of hydrodynamic force exerted on the body. These vortices also result in the generation of some peculiar and fascinating free-surface signatures in the near field, a finding that is of relevance in remote-sensing technology applied to deciphering the wave signature of submerged ocean vehicles and tethered mines.

### VISCOUS-FLOW FORMULATION

In this paper, the governing equations and all physical quantities are presented in a nondimensional form, with fundamental variables corresponding to density of fluid  $\rho^*$ , acceleration of gravity  $g^*$ , and radius of the cylinder  $R^*$ . We attach a \* super-

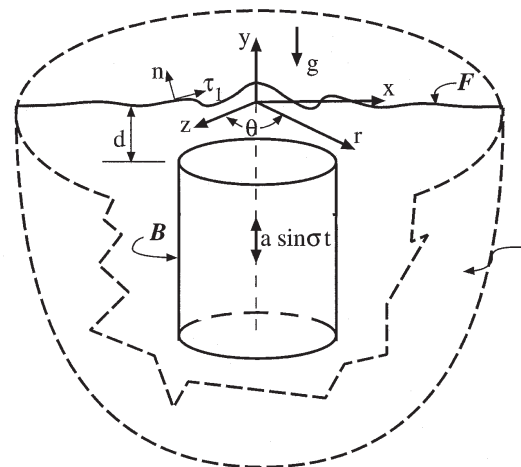


Fig. 1 Heave oscillation of submerged vertical cylinder: coordinate system and notations

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