

Second-Order Approximate Solution of Nonlinear Wave Diffraction Due to Vertical Cylinder Array

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ABSTRACT

A second-order analytical solution for nonlinear wave diffraction due to multiple vertical cylinders has been derived. Laboratory experiments have been conducted to investigate the validity of the obtained solution. By examining theoretical and experimental results of the water surface profiles, it has been confirmed that the attained solution is good enough to simulate the water surface profile, although it cannot exactly satisfy the equation of continuity because of an approximation.

INTRODUCTION

A second-order analytical solution for nonlinear wave diffraction caused by an isolated surface piercing cylinder was obtained by Hunt and Baddour (1981), Kriebel (1990) and Chau and Eatock Taylor (1992). Kim and Yue (1989) also derived the solution for axisymmetric body. The corresponding solution in the case of multiple cylinders has not been derived yet, although some researchers have calculated the wave forces on them with use of the radiation potential (Ghalayini and Williams, 1991). The wave profile, in particular runup height on the cylinder surface, may be enlarged due to the nonlinear interaction of the diffracted waves generated by each cylinder. Thus, a theory which can evaluate the nonlinear behavior of diffracted waves is necessary indeed.

The authors have derived a second-order analytical solution of the velocity potential for the diffracted wave from an isolated cylinder without using Green's function (Sanada et al., 1996). In this study, the solution for the isolated cylinder has been modified to include the case of multiple cylinders, employing the same method developed by Linton and Evans (1990). The accuracy of the proposed theory has been validated by laboratory experiments.

ANALYTICAL DERIVATION

In this study, we have considered the case of N bottom-seated vertical surface piercing cylinders which are located at $(x, y) = (X_j, Y_j)$ ($j = 1 \sim N$) in the sea area at constant water depth (d), where (x, y) are the horizontal Cartesian coordinates. The radius of each cylinder is a_j ($j = 1 \sim N$), as shown in Fig. 1, and the incident wave which propagates in the direction of β has a height of H_1 and a frequency of σ . The local cylindrical coordinate system (r_j, θ_j, z) with the origin at the center of each cylinder was used; let R_{jk} be the distance from cylinder j to cylinder k , and δ_{jk} and δ_{kj}

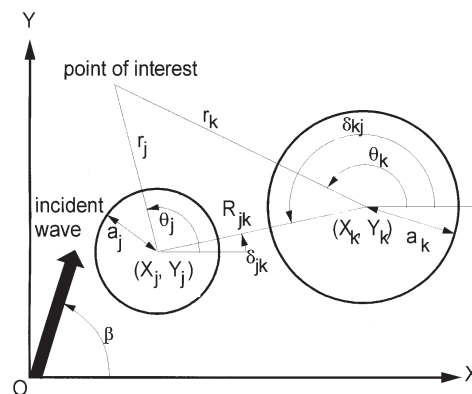


Fig. 1 Definition sketch

the angles measured from the x -axis to R_{jk} at both ends. (See Fig. 1.)

Governing Equations and Boundary Conditions

The assumption of irrotational motion of incompressible and inviscid fluid makes it possible to apply the velocity potential for describing the wave field. The velocity potential around the cylinder k is denoted by $\Phi^k(r_k, \theta_k, z; t)$ and can be expanded by the perturbation series with ϵ . ($\epsilon = k_1 H_1$; k_1 is the wave number of the first order approximation.) The result up to the second order is given by Eq. 1:

$$\Phi^k(r_k, \theta_k, z; t) = \text{Real} \left(\epsilon \phi_1^k(r_k, \theta_k, z) e^{-i\sigma t} + \epsilon^2 \phi_2^k(r_k, \theta_k, z) e^{-2i\sigma t} + \overline{\epsilon^2 \phi_2^k(r_k, \theta_k, z)} + \epsilon^2 \delta_1^{(2)} t \right) \quad (1)$$

where $\overline{\phi_2^k(r_k, \theta_k, z)}$ is the velocity potential corresponding to the time independent component. The last term in Eq. 1 corrects the mean surface level change (e.g., Ogilvie, 1983; Eatock Taylor and Hung, 1987). Although the third term in Eq. 1 gives the steady component of the velocity, it does not affect the wave profile, pressure and forces in the second-order approximation. Thus, in this study, $\phi_2^k(r_k, \theta_k, z)$ is disregarded, and only $\phi_1^k(r_k, \theta_k, z)$ and

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Received February 1, 1994; revised manuscript received by the editors March 1, 1996; revised manuscript received by the editors February 25, 1997. The original version (prior to the final revised manuscript) was presented at the Sixth International Offshore and Polar Engineering Conference (ISOPE-96), Los Angeles, USA, May 26-31, 1996.

KEY WORDS: Diffraction theory, nonlinear wave diffraction, cylinder array.