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Fixed Finite Element Model of Heat Transfer with Phase Change — Part I. Theoretical Formulation and Numerical Algorithm

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ABSTRACT

The paper focuses on the theoretical formulation and the numerical algorithm of the heat transfer with phase change for fixed domain. The law of conservation of energy for the media with discontinuities forms the basis for obtaining a weak formulation of the problem including the most of possible thermal boundary conditions. The formulation presented enables one to obtain the weak form of the problem with implicitly introduced boundary conditions of discontinuous temperature gradient at the phase change interface. The Euler backward implicit time integration method together with equilibrium iterations is used to predict the transient response. The treatment of the phase change employs the concept of normality of temperature gradient to the phase change interface. The 2-D fixed mesh of the finite elements applied in the discretization of the analysed problem is combined with the 1-D FEM model of the moving phase change interface. A special procedure is used to build solution matrices for discontinuous elements intersected by the moving phase change boundary.

INTRODUCTION

The problem of the heat transfer with phase change occurs in various fields of technological interest such as geotechnics (soil freezing), the glass and steel industry, biology (freezing of tissues), the food industry and others. The importance of the phase change problem has resulted in a large number of different methods developed to analyse it; extensive reviews of methods currently in use can be found in the papers of Ockendon and Hodgkins (1975), Wilson, Solomon and Boggs (1978), Salcudean and Abdullah (1988), and Dalhuijsen and Segal (1986).

The heat transfer without phase change can be analysed by means of different numerical methods, which transform the differential equation governing the heat transfer into the system of algebraic equations. The serious difficulties appear in standard methods when the trial functions have to satisfy boundary conditions at the phase change interface moving in time.

One way to solve such problems is to use a deforming grid formulation (O'Neill and Lynch, 1981; Hogge and Gerrekens, 1983; Lynch, 1982; Kececioglu and Rubinsky, 1989; Elliot and Ockendon, 1982). The intrinsic feature of this method is that the phase change interface always lies on one side of some determined elements. According to the translation of the interface, the grid moves in such a way that it remains the same set of elements in the solid and liquid phases. The governing equations of each phase are solved separately. The phase change front is directed according to the balance condition of the latent heat and fluxes across the front. This type of method requires a starting solution which must be estimated by another technique. It is also worth adding that this method cannot be used to model the simultaneous movement of multiple fronts or in the problems with appearing and disappearing phases.

The deforming grid is used, for instance, by Kececioglu and Rubinsky (1989). Their research is connected with porous media that involve coupled heat and mass transport that yielding a steep continuous moving front and an abrupt discontinuous moving phase change interface. They propose a variational method to derive the finite element equations for the physical variables and the movement multiple dimensions of the deforming grid. It is obtained from the weighted least squares' residuals of a coupled system of partial differential equations. Their formulation of the mixed finite element scheme employs the simultaneous piecewise continuous interpolation of primary variables and gradients of the primary variables. The obtained results for the fluxes are much more accurate than those obtained from the methods based on piece-wise continuous interpolation of the primary variables only.

Another important group of methods dealing with analysis of the geometry of moving interface (Wilson, Solomon and Boggs, 1978; Elliot and Ockendon, 1982; Comini et al., 1974; Morgan, Lewis and Zienkiewicz, 1978; Bell and Wood, 1983; Tacke, 1985; Tamma and Railkar, 1988; O'Neill, 1983; Rolph and Bathe, 1982; Roose and Storrer, 1984; Crivelli and Idelsohn, 1986) can be classified as the fixed domain methods. This group uses the weak formulation of the problem and in this way the moving interface is involved in the integral form of the equation of heat transfer. Within this group there are two possible formulations, depending on the choice of the unknown variable, i.e., enthalpy or temperature.

The enthalpy formulation utilises the fact that the enthalpy varies continuously, also across the moving front. After the solution for nodal enthalpies is obtained, the temperature distribution can be determined from the temperature-enthalpy relation. However, a spurious constant temperature region appears as a result of the thermal plateau.

Application of the enthalpy formulation used by Comini et al. (1974) enables one to approximate latent heat effect by means of the heat capacity distributed over a small range of temperatures

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