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A Numerical Model for Fluid-Particle Flows

F. Lalli* and A. Di Mascio
Istituto Nazionale per Studi ed Esperienze di Architettura Navale, INSEAN, Rome, Italy

ABSTRACT

In this paper a numerical algorithm for fluid-particle flow computation is presented. The mathematical formulation is based on the averaged continuum model, in which the effects of particles are taken into account in terms of an effective viscosity. The dispersed phase equation closure is based on sedimentation and shear-induced self-diffusion effects. The present work is the first step in the development of a general model for the simulation of the interaction between waves or currents and bottom sediment. Namely, the proposed approach allows the study of sediment transport and the evolution of the bottom shape without the need for curvilinear coordinate systems and related step-by-step regridding. In fact, pure liquid regions, suspension regions (more or less concentrated) as well as bottom sediment are studied by a unique model with a proper effective viscosity (hindrance effect and Bingham viscoplastic model). Preliminary numerical results have been obtained for 2D Bingham flow in a driven cavity by a finite difference method.

INTRODUCTION

Sediment transport is one of the most important problems in coastal and river engineering; though it has been extensively studied for several years, yet it is not possible to predict with reasonable accuracy the evolution of a coast affected by waves and currents. Even in simple situations, the relationship between the fluid velocity and the mass transport of sediment is not clear, because of the dynamic interaction between the flow and the particles. Moreover, the bottom behaves like a free surface, with the generation of waves and the related flow perturbation. Sediment transport is therefore a strongly nonlinear problem, showing a negative feedback between the flow and the moving particles.

The fluid mechanical theory of the dynamical interaction of a collection of particles and the surrounding fluid has been developed (Davidson and Harrison, 1971) by representing the system as two interpenetrating and interacting continua. As might be expected, the equations of motion for the model stated in these terms are considerably more complicated than for the case of a single fluid. In particular it is necessary to obtain constitutive relations for the stress tensors of both phases, as well as for the interaction force, and to solve the system of equations (in 3-D, 8 scalar equations) with suitable initial and boundary conditions.

For practical purposes it is necessary to seek some simplifications, so that it can be described in terms comparable with the continuum mechanics of a single fluid. In several practical applications the details of the flow and the motion of particles are not required, whereas it is important to know the collective particle motion; a single fluid model, in which the effect of the particulate phase is accounted for in terms of an effective viscosity, can be a rather effective tool. The mechanics of a collection of particles interacting with the surrounding fluid can therefore be studied regarding the two phases as jointly forming a single pseudocontinuum. Many papers about this approach can be found in the literature, most of which are concerned with the relation between the rheology of suspensions and the properties of the fluid and the particles; among these, one could mention the classic paper by Einstein (1906) concerning the viscosity of dilute suspensions, and more recent works (Krieger, 1972; Happel and Brenner, 1986; Leighton and Acrivos, 1987) in which the results obtained by Einstein are extended for higher concentrations.

In this paper the numerical solution of a model based on this kind of approach is presented. The fluid is assumed to be Newtonian in regions of dilute suspensions, with viscosity related to concentration, whereas, for high amount of suspended particles, the intergranular forces are represented by a superposition of hindrance effect and shear-dependent behaviour (Bingham model). Relative motion is modeled by introducing shear-induced particle migration effects (Leighton and Acrivos, 1987) together with the more classic settling velocity.

A preliminary numerical example concerning the simulation of a uniform-density Bingham fluid in a driven cavity has been performed, to verify the capability of the algorithm to deal with strong viscosity variations flows; this test gives rather encouraging results, in that the generation of regions of rigid motion inside the fluid domain can be computed with reasonable accuracy. The object of a future work will be the direct simulation of the interaction of gravity waves with the bottom sediment.

MATHEMATICAL FORMULATION

Basic Equations

Fluid-particle interaction is studied by means of a pseudocontinuous medium; the mathematical model can be outlined as follows:

$$F_{1,u}(l_{CAN}/c \tag{1}$$

$$\frac{\partial \rho}{\partial t}+\frac{\partial(\rho v_i)}{\partial x_i} = 0 \tag{2}$$

$$\rho\left(\frac{\partial v_i}{\partial t}+v_j\frac{\partial v_i}{\partial x_j}\right) = -\frac{\partial p}{\partial x_i}+\chi(\rho_p-\rho_w)g_i$$

*ISOPE Member.

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KEY WORDS: Fluid-particle flows, numerical model, diffusion, waves, current, sediment.