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Extension of Free-Surface Green's Function Multipole Expansion for Infinite Water Depth Case

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This paper presents new developments on the multipole expansion of the infinite water depth free-surface Green's function, in the scope of wave farm simulation. The multipole expansion of the Green's function and its derivatives have been extended to be used in a 3D fast multipole algorithm. Previous restrictions over the use of the multipole expansion are proven to be unnecessary. Extensive validation is provided by evaluating this function over a large range of parameters. The influence of the fast multipole algorithm parameters on the accuracy of the multipole expansion and on the CPU time are then investigated.

INTRODUCTION

Wave Energy Converters (WEC) are dedicated to be deployed in large arrays of typically 10 to 100 systems. An optimal spacing between the devices can help achieve the objectives of improving the energy production and smoothing the overall power output. Simulation is necessary to determine how the farm should be organized, and to investigate interactions effects.

The resolution of the radiation/diffraction problems for a large array of systems using Boundary Element Methods (BEM) involves building and solving large, dense linear systems, requiring a $O(N_{\text{panels}}^3)$ complexity. The challenge is to carry out each simulation fast enough to investigate different parameters in a reasonable amount of time: spacing between devices, wave parameters, bathymetry. A well-known solution for accelerating the BEM is the implementation of a General Minimum RESidual (GMRes) iterative solver with a Fast Multipole Algorithm (FMA) for the fast calculation of matrix-vector products (MVP). This way the computation complexity can be reduced from $O(N_{\text{panels}}^3)$ to $O(N_{\text{panels}})$.

The FMA is based on the multipole expansion of the free-surface Green's function. For the constant depth case, the Green's function is described as a series of terms containing the modified Bessel function of the second kind K_0 (Newman, 1985). Using Graf's addition theorem, the multipole expansion has been derived by Utsunomiya and Watanabe (2002). Combining this expansion, the Higher Order Boundary Elements Method (HOBEM) and FMA, the hydrodynamic responses of a Very Large Floating Structure (VLFS) have been investigated. In Teng and Gou (2006), the results of the combination of the Constant Panel Method (CPM) or the HOBEM and the FMA are compared to analytical solutions for a floating box and a floating cylinder. In Gou and Teng (2008), the hydrodynamic interactions between 3 ships closely spaced have been studied.

Recently an expansion for the free-surface Green's function has been developed for the infinite water depth case (Utsunomiya and Okafuji, 2007) and applied to the case of a VLFS. This formulation is appropriate for describing a wave farm, which would ideally be situated in large depth, to avoid energy losses in the incident waves due to bathymetry effects. In the case of shallow water, it is still possible to consider a complex seabed, represented as an independent, non moving body.

The present paper is a continuation of this work: It extends the expansion formulation to make possible its use in a 3D FMA, and the calculation of the normal derivatives of the Green function is described. The multipole expansion and the translation operators are extensively tested. The final objective here is to integrate these formulations and the FMA in an in-house diffraction/radiation code, Aquaplus (Delhommeau, 1993).

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KEY WORDS: Free-surface Green's function, Boundary Element Method (BEM), Fast Multipole Algorithm (FMA), multipole expansion, wave farm, Wave Energy Converters.

FORMULATION

Boundary Element Problem

Here we introduce the resolution of the diffraction/radiation problem by the software Aquaplus, which is based on the BEM. The water is modeled as inviscid and incompressible. The fluid velocity is the gradient of a potential ϕ. The corresponding boundary problem is:

$$\Delta\phi = 0 \quad \text{in all the fluid domain} \tag{1}$$

$$\frac{\partial\phi}{\partial n} = 0 \quad \text{on the seabed} \tag{2}$$

$$\frac{\partial\phi}{\partial n} = \vec{V}_i \cdot \vec{n} \quad \text{on the surface } S_i \text{ of the body } i \tag{3}$$

$$\frac{\partial^2\phi}{\partial t^2} + g\frac{\partial\phi}{\partial z} = 0 \quad \text{on the free surface} \tag{4}$$

where g is the gravitational acceleration, and $\vec{V}_i$ the velocity on the surface of the body i. Defining:

$$\Phi = \text{Re}[\phi e^{(-i\omega t)}] \tag{5}$$

$$\Phi(M) = \iint_{\sum_i S_i} \sigma(M') \cdot G(M, M')dS \tag{6}$$

and applying the second Green's formula leads to the following integral equation:

$$\frac{\sigma(M)}{2} - \frac{1}{4\pi}\iint_{\sum_i S_i} \sigma(M')\,\overrightarrow{\text{grad}}\,G(M;M') \cdot \vec{n}_{xyz}dS = \vec{V}_i \cdot \vec{n}_{xyz} \tag{7}$$