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Dynamic Response of an Ice-Covered Fluid to a Submerged Impulsive Point Source

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Flexural-gravity waves generated by an explosion under an ice sheet are considered. The underwater explosion is modeled by a submerged point mass source. It is assumed that water is an ideal incompressible fluid, and that the motion of the liquid is potential. The ice cover is modeled by an initially unstrained viscoelastic, homogeneous, isotropic plate. The analysis is carried out by using Fourier and Laplace integral transforms. The effects on the ice plate deflection of the basin depth, ice plate thickness and submergence depth of the impulsive source are analyzed.

INTRODUCTION

Ice blasting is recognized as an effective way of destroying ice covers. When the ice cover is ruptured or destroyed by an explosion, it is known that an underwater explosion is more effective (Kozin, 2007). The effect of a singular impulsive load on an ice sheet has been thoroughly investigated by Kerr (1976), Fox (1993) and Kheisin (1967), while Lu and Dai (2008) have studied the dynamic responses of an ice-covered fluid of finite depth due to fundamental singularities, particularly the submerged source. Two kinds of unsteadiness were considered: instantaneous and time-harmonic singularities. Solutions were obtained in integral form, but the problem was solved asymptotically only for large time and distance.

In the authors' opinion, the introduction of a viscoelasticity into the equation for the ice plate permits the numerical evaluation of the modified integral. Thus the aim of this paper is to solve the problem of viscoelastic deflection of the ice plate floating on water of finite depth with an impulsive mass source immersed in the fluid. The solution of the problem will facilitate the search for better techniques for ice-cover destruction or rupturing by means of blasting.

GENERAL MATHEMATICAL FORMULATION

We consider an initially unstrained, homogeneous, isotropic, viscoelastic ice plate lying on an elastic liquid base. The plate is in the state of rest and is subjected to a shock pulse from a submerged point mass source at time $t = 0$. The coordinate system is arranged as follows: The Oxy plane coincides with the unperturbed ice-water interface, and the Oz axis is directed vertically upwards. The mass impulsive source is situated at point $(0, 0, -d)$. The flow of fluid of density ρ_2 is assumed to be irrotational.

Viscoelasticity is assimilated by using a Kelvin-Voigt spring-dashpot arrangement (Kheisin, 1967; Freudenthal and Geiringer, 1962). Consequently, the system to be solved is:

$$\frac{Gh^3}{3}\left(1+\tau_K\frac{\partial}{\partial t}\right)\nabla^4\zeta+\rho_1 h\frac{\partial^2\zeta}{\partial t^2}+\rho_2 g\zeta+\rho_2\left.\frac{\partial\Phi}{\partial t}\right|_{z=0}=0; \qquad (1)$$

$$\left.\frac{\partial\Phi}{\partial z}\right|_{z=0}=\frac{\partial\zeta}{\partial t}, \quad \left.\frac{\partial\Phi}{\partial z}\right|_{z=-H}=0, \qquad (2)$$

$$\Phi(x,y,z,t)=\varphi^S(x,y,z,t)+\varphi^R(x,y,z,t), \qquad (3)$$

$$\varphi^S(x,y,z,t)=-\frac{Y_0 Y(t)}{4\pi\sqrt{x^2+y^2+(z+d)^2}}, \qquad (4)$$

$$\zeta|_{t=0}=0, \quad \left.\frac{\partial\zeta}{\partial t}\right|_{t=0}=0, \qquad (5)$$

where $\varsigma(x, y, t)$ is the vertical deflection of the ice-water interface; G is the elastic shear modulus of the ice, $G = 0,5E/(1+\nu)$; E is the Young's modulus; ν is the Poisson's ratio; $h(x, y)$ is the plate thickness; $\rho_1(x, y)$ is the ice plate density; τ_K is the strain relaxation time for ice sometimes called the delay (Kheisin, 1967; Freudenthal and Geiringer, 1962); $\Phi(x, y, z, t)$ is the fluid velocity potential function satisfying Laplace's equation $\Delta\Phi = 0$; $\varphi^S(x, y, z, t)$ is the velocity potential due to the simple source in an unbounded domain; Y_0 is the constant strength of the simple source; $Y(t)$ is an impulsive source function of time; $\varphi^R(x, y, z, t)$ is a regular component of the potential function $\Phi(x, y, z, t)$ that describes the velocities of the wave motion; d is the submergence depth of source; $H = H_1 - b$; where H_1 is the basin depth; and $b = \rho_1 h/\rho_2$ is the ice immersion depth in static equilibrium.

In the manner of Lu and Dai (2008), we incorporate the Dirac delta function for impulsive function $Y(t)$:

$$Y(t)=\delta(t). \qquad (6)$$

In addition, we also consider another impulsive function:

$$Y(t)=\begin{cases}\dfrac{a}{2}\sin(at), & 0\le t\le\dfrac{\pi}{a},\\ 0, & t>\dfrac{\pi}{a}\end{cases} \qquad (7)$$

which will be referred to as the sinusoidal function in this paper.

ANALYTICAL SOLUTION

By analogy with Lu and Dai (2008a), the function ζ is found by using Fourier and Laplace integral transforms as:

$$\zeta(r,t)=\frac{Y_0}{4\pi}\int_0^\infty J_0(kr)fM\,k\,dk \qquad (8)$$

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