

# Wave Impact Pressure Calculations by Improved SPH Methods

Abbas Khayyer and Hitoshi Gotoh  
Department of Urban and Environmental Engineering, Kyoto University  
Katsura Campus Nishikyo-ku, Kyoto, Japan

**This paper presents improved Incompressible SPH (ISPH) methods for the prediction of wave impact pressure, more specifically, impact pressure due to sloshing waves. The first improvement is the employment of a corrective function for enhancing angular momentum conservation. The second improvement applies a higher-order source term that is derived from a higher-order differentiation for a less fluctuating and more accurate pressure calculation. The third improvement proposes a new criterion for a more accurate assessment of free-surface particles in a particle-based calculation. The enhanced performance of improved ISPH methods in predicting wave impact pressure has been shown by simulating 4 cases of sloshing waves induced by sway excitations (Kishev et al., 2006) and rotational ones (Delorme et al., 2009).**

## INTRODUCTION

With the trend towards the LNG industry's massive expansion and development, the sloshing of LNG in partially filled tanks has attracted many researchers in the marine and offshore community. In this context, one of the major concerns is the prediction of sloshing-induced impact pressure which can cause critical damage to the LNG carriers.

During the past 3 decades, a great number of experimental (e.g. Berg, 1987; Akyildiz and Unal, 2005; Bunnik and Huijsmans, 2009) and theoretical (e.g. Abramson, 1966; Popov et al., 1992; Ibrahim, 2005) studies have been devoted to sloshing flows and sloshing-induced impact loads.

In parallel to the experimental and theoretical investigations, extensive numerical simulations of sloshing flows have been carried out, based on solving the continuity and Navier-Stokes equations (e.g. Aus Der Wiesche, 2003), or the potential flow equations (e.g. Firouz-Abadi et al., 2008). Nevertheless, because a violent sloshing flow characterized by free-surface breaking and fluid fragmentations can no longer be assumed to be irrotational, potential flow-based methods do not seem to be applicable in such a case. On the other hand, grid-based Navier-Stokes solvers require an appropriate mathematical treatment of the free surface for simulation of a violent sloshing flow. Kim (2001) and Lee et al. (2007) applied the SURF and VOF schemes in their finite difference and finite volume-based calculations of sloshing waves, respectively. To attenuate the numerical diffusion arising from a grid-based discretization of the continuity and the Navier-Stokes equation, and to consider the effect of air entrainment, Kishev et al. (2006) applied the CIP method (Yabe et al., 2001) for simulations of violent sloshing flows.

When dealing with violent free-surface fluid flows accompanied by breaking and fragmentations (as in the case of a violent sloshing flow), a Lagrangian grid-less method, namely, a particle method, appears to be a suitable candidate. Because of their fully Lagrangian treatment of advection terms in both the Navier-Stokes and the free-surface evolution equations, particle methods are free from the numerical diffusion (corresponding to

the advection terms). Further, due to their grid-less feature, these methods can easily handle large deformations and fragmentations. Hence, particle methods provide a substantial potential for simulation of violent sloshing flows and their induced impact pressure.

The Smoothed Particle Hydrodynamics (SPH; Gingold and Monaghan, 1977; Lucy, 1977) is one of the earliest particle methods invented for modeling astrophysical phenomena. Since then, it has been widely extended to applications to the problems in continuum solid and fluid dynamics, including free-surface hydrodynamic fluid flows. Despite its capability and wide range of applicability, the SPH method has a few shortcomings that may considerably affect its accuracy and performance. Such shortcomings are mainly brought about by the interpolation features of SPH, that is, local kernel-based interpolations on the basis of moving calculation points by relatively simple differential operator models. Among the major shortcomings associated with the SPH method are the non-exact conservation of momentum (Bonet and Lok, 1999; Khayyer et al., 2008), the lack of interpolation completeness (Liu et al., 1993) and the existence of spurious pressure fluctuations (Colagrossi and Landrini, 2003; Gotoh et al., 2005).

The accuracy and performance of particle methods, including the SPH method, have been enhanced by applying techniques correcting the kernel function itself and/or its gradient in order to improve the completeness of kernel interpolants and/or to enhance the conservation of momentum, as in the case of the Corrected SPH (CSPH; Bonet and Lok, 1999), Corrected Incompressible SPH (CISPH; Khayyer et al., 2008) or Corrected Moving Particle Semi-implicit (CMPS; Khayyer and Gotoh, 2008) methods.

To resolve the problem of artificial pressure fluctuation in their Weakly Compressible SPH (WCSPH) simulations, Colagrossi and Landrini (2003) re-initialized the density field at distinctive calculation time steps by employing a 1st-order accurate interpolation scheme via the application of a moving-least-square kernel approximation. A more accurate interpolation scheme enhances the consistency of the mass-density-occupied area, thus resulting in a less fluctuating and more accurate source term of pressure equation (equation of state). Accordingly, a less fluctuating and more accurate pressure field will be obtained. Delorme et al. (2009) applied the same approach to achieve improved sloshing-induced impact pressures in their WCSPH simulations of violent sloshing flows. On the contrary, the Incompressible SPH (ISPH) method (Shao and Lo, 2003) employs a Poisson Pressure Equation (PPE) in which the pressure is a direct function of the time rate of change of density rather than the density itself. Thus, to attain

---

Received March 16, 2009; revised manuscript received by the editors August 3, 2009. The original version was submitted directly to the Journal.

KEY WORDS: Particle method, SPH, Improved Incompressible SPH, wave impact pressure, sloshing.