

# Lagrangian Particle Method as Advanced Technology for Numerical Wave Flume

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**The particle method, which is a solver of the Navier-Stokes equation without using a computational grid, has excellent robustness in analyzing a violent water-surface change accompanied with a fragmentation and coalescence of water. Thus the particle method is an optimum tool for the analysis of the process of wave breaking and runup. Here are outlined the calculation fundamentals of the particle method. The state-of-the-art of the particle method is briefly introduced, including highly precise particle methods, by improving momentum conservation in discretization of governing equations and the new methods for control of pressure fluctuation. Finally, we show as prospective studies on the particle method a few of the significant issues for promoting the substantial contribution of the particle method to a numerical wave flume, which is the computer-aided resistive design tool of coastal structures against wave action.**

## INTRODUCTION

In order to describe nonlinear wave transformation, the Boussinesq (1872) theory and its extension (e.g. Nwogu, 1993) are popular and reliable. However, these models are unable to describe wave breaking directly, because they are derived under the assumption of potential flow, namely irrotational flow, while neglecting viscosity. In extended models of the Boussinesq theory, wave breaking was described empirically as an energy damping effect using ad hoc energy dissipating schemes (e.g. Svendsen, 1984).

Computations based on analytical governing equations of wave motion, such as the Boussinesq equation, must be replaced by numerical solutions of the Navier-Stokes equation in order to analyze a wave breaking and runup process without ad hoc sub-models of energy dissipation.

In the wave breaking phenomena, there exists one more difficulty: the existence of multiple connected flow domain, or topological change of the free surface, due to the plunging jet striking the toe of the wave. Under this condition, Lagrangian grid methods, which track a free surface by using a moving grid, break down.

The Marker-And-Cell (MAC) method (Harlow and Welch, 1965), which captured the water surface by tracking markers existing in its vicinity, was the first computational approach to overcome this difficulty. The Volume Of Fluid (VOF) method (Hirt and Nichols, 1981) was proposed to improve the computational efficiency of water-surface tracking of the MAC method. Currently, numerical diffusion arising from fixed-point interpolations of advection terms in the VOF-function transport equation is recognized as being a significant drawback in its reliability. For the grid-based Eulerian solvers of the Navier-Stokes equation, a sophisticated scheme, namely the CIP method (Yabe et al., 2001), has been proposed to attenuate the numerical diffusion. Hu and

Kashiwagi (2004) applied the CIP method to wave impact calculations and obtained quite satisfactory results.

A more straightforward and efficient approach to a violent flow with complicated free-surface behavior is the Lagrangian gridless method, namely, the particle method, in which the particle is used for both a calculation point and a marker for water-surface tracking. Because of its Lagrangian nature, the particle method is able to analyze flow field without the numerical diffusion. Fundamentals of the particle method are outlined below.

## PARTICLE METHODS

There are 2 popular particle methods: the Smoothed Particle Hydrodynamics (SPH) method (Gingold and Monaghan, 1977; Lucy, 1977) and the Moving Particle Semi-implicit (MPS) method (Koshizuka and Oka, 1996). The SPH method was developed as a solver of compressible flow to model the dynamics of galaxies in astrophysics. Later on, the Weakly Compressible version (WCSPH) was proposed by Monaghan (1994) for analyzing incompressible free-surface fluid flows.

The MPS method was developed for the simulation of incompressible fluid flows. There are 2 fundamental differences between the SPH and the MPS methods: One is the role of the kernel for local integration, and the other is the computational scheme.

In the particle method, vector differential operators, such as gradient, divergence and Laplacian, are modeled by the local integration of physical properties using a radial kernel. In the SPH method, differential operation is described by differentiation of the kernel, while in the MPS method all the vector differential operators are modeled one by one, with the kernel playing a weight function role.

In the SPH and the WCSPH, explicit schemes are generally used for computation, while in the MPS the pressure field is solved by an implicit scheme. Shao and Lo (2003) proposed the Incompressible SPH (ISPH) method by introducing the implicit scheme of the MPS method for solving the Poisson Pressure Equation (PPE) using the SPH kernel for differential operations.

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