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Submarine Moving Close to Ice Surface Conditions

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This paper deals with the submarine in ice conditions. The ice cover is modeled by a floating Euler-Bernoulli plate. The hydrodynamic problem of submarine motion is modeled by a point source which steadily moves in water under the floating ice plate. The analysis is carried out by using the analytical methods theory of the function of complex variables and integral transformations. Then the integral for ice plate deflection is evaluated numerically. The given solution is analyzed as a function of submarine velocity, ice thickness and submergence depth.

INTRODUCTION

Waves are known to appear on the surface of the water when a solid body is moving in the water, with the waves moving in the direction of the body motion. The floating ice cover causes changes in the water surface boundary conditions. Thus, corresponding changes can be expected in the characteristics of the wave formation.

The investigations by Kheisin (1967) should be mentioned among the theoretical works dedicated to the problem. Kheisin's paper considered the 2-dimensional problem of the motion of a point vortex under a layer of broken ice. There it was found that the broken ice produced only minor changes to the gravitational waves caused by the motion of the submerged body. The 1994 article by Kozin and Onishchuk is devoted to model experiments proving the possibility of the moving submarine's destruction of the ice plate.

The aim of this paper is to study the form of flexural waves caused by the steady motion of a point source submerged beneath an ice sheet.

MATHEMATICAL STATEMENT

To solve the problem of solid body motion under the surface of water covered by floating ice, we use the procedure of solving an analogous problem without ice conditions (Sretensky, 1977).

We consider the steady straight motion of the point source Q (strength q) with velocity V under the surface of infinitely deep water of density ρ_2. An Euler-Bernoulli plate of density ρ_1 and thickness h is floating on the surface of the water. The Cartesian coordinate system $Oxyz$ connected with source Q is arranged as follows: The Oxy plane coincides with the unperturbed ice-water interface, the x direction coincides with the direction of the source motion, and the Oz axis is directed vertically upwards. The liquid motion is assumed to be irrotational.

In the moving coordinate system, the velocity potential function $\Phi(x, y, z)$ is assumed to be independent of time. It represents the sum of:

- the velocity potential of the unperturbed water flow possessing velocity $-V$;
- the velocity potentials of the source $Q(0, 0, -H)$ and sink $Q'(0, 0, H)$ (here H is the submergence depth); and,
- the velocity potential of the wave motion.

The velocity potential $\Phi(x, y, z)$ is given in the form:

$$\Phi(x, y, z) = -Vx - \frac{q}{4\pi\sqrt{x^2+y^2+(z+H)^2}} + \frac{q}{4\pi\sqrt{x^2+y^2+(z-H)^2}} + \varphi(x, y, z), \tag{1}$$

where the function $\varphi(x, y, z)$ describes the velocities of the wave motions. To define this function we consider the boundary condition on the water surface (Kheisin, 1967; Squire et al., 1995):

$$\frac{D}{\rho_2 V^2}\nabla^4\zeta + \frac{\rho_1 h}{\rho_2}\frac{\partial^2\zeta}{\partial x^2} + \frac{g}{V^2}\zeta - \frac{1}{V}\frac{\partial\Phi}{\partial x} = 0 \qquad (z=0) \tag{2}$$

and the linearized kinematic condition on the ice-water interface:

$$\left.\frac{\partial\Phi}{\partial z}\right|_{z=0} = -V\frac{\partial\zeta}{\partial x}. \tag{3}$$

Here ζ is the vertical deflection of the ice plate; $D = Eh^3/12(1-\nu^2)$ is the uniform rigidity of the plate; E is Young's modulus; and ν, Poisson's ratio.

By applying the kinematic condition Eq. 3, we can write the boundary condition Eq. 2 in the form:

$$\frac{D}{\rho_2 V^2}\frac{\partial}{\partial z}\nabla^4\Phi + \frac{\rho_1 h}{\rho_2}\frac{\partial^3\Phi}{\partial z\partial x^2} + \frac{g}{V^2}\frac{\partial\Phi}{\partial z} + \frac{\partial^2\Phi}{\partial x^2} = 0 \qquad (z=0) \tag{4}$$

Substituting expression Eq. 1 for $\Phi(x, y, z)$ into Eq. 4 gives:

$$\begin{aligned}&\frac{D}{\rho_2 V^2}\frac{\partial}{\partial z}\nabla^4\varphi + \frac{\rho_1 h}{\rho_2}\frac{\partial^3\varphi}{\partial z\partial x^2} + \frac{g}{V^2}\frac{\partial\varphi}{\partial z} + \frac{\partial^2\varphi}{\partial x^2}\\ &= \frac{qDH}{4\pi\rho_2 V^2}\\ &\quad\cdot\left(\frac{-1890\left(x^2+y^2\right)^2}{\left(x^2+y^2+H^2\right)^{11/2}} + \frac{1680\left(x^2+y^2\right)}{\left(x^2+y^2+H^2\right)^{9/2}} - \frac{240}{\left(x^2+y^2+H^2\right)^{7/2}}\right)\\ &\quad + \frac{q\rho_1 hH}{4\pi\rho_2}\left(\frac{-30x^2}{\left(x^2+y^2+H^2\right)^{7/2}} + \frac{6}{\left(x^2+y^2+H^2\right)^{5/2}}\right)\\ &\quad - \frac{qgH}{2\pi V^2\left(x^2+y^2+H^2\right)^{3/2}} \qquad (z=0)\end{aligned} \tag{5}$$

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