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Code Verification of Unsteady Flow Solvers with Method of Manufactured Solutions

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This paper presents manufactured solutions for code verification of Navier-Stokes solvers for 2-dimensional, unsteady flows of an incompressible fluid. Two solutions are proposed: A periodic solution, and one that evolves to a steady state. A first application of the manufactured solutions is presented for an unconventional implicit method that solves the continuity equation without any transformation.

INTRODUCTION

The maturing of CFD codes for practical calculations in engineering implies the need to establish the credibility of the results by Verification & Validation (V&V). A clear and simple definition of V&V is given by Roache (1998): Verification is a purely mathematical exercise that intends to show that we are "solving the equations right," while validation is a science/engineering activity that intends to show that we are "solving the right equations."

Verification is in fact composed of 2 different activities, Code Verification and Verification of Calculations (Roache, 1998):

• Code Verification intends to verify that a given code solves correctly the equations of the model that it contains by error evaluation.

• On the other hand, Verification of Calculations intends to estimate the error of a given calculation, for which in general the exact solution is not known. Verification of Calculations should be preceded by Code Verification, as underlined by the conclusions of the first workshop on uncertainty analysis, held in Lisbon in 2004 (Eça and Hoekstra, 2004; Eça, Hoekstra and Roache, 2005).

In general, unsteady flow calculations are time consuming, and so it is not very common to see verification exercises of unsteady flow solvers. However, the need to address Code and Calculation Verification in unsteady flow solvers is unquestionable. Code Verification requires the knowledge of the exact solution, which is not easy to obtain in unsteady flows. The Method of Manufactured Solutions (MMS) (Pelletier and Roache, 2002; Oberkampf, Blottner and Aeschliman, 1995; Turgeon and Pelletier, 2001; Turgeon and Pelletier, 2002; Knupp and Salari, 2002; Roache, 2002; Eça et al., 2006) is a viable alternative for performing Code Verification of unsteady flow solvers. In the MMS, a continuum solution is first constructed, i.e. one specifies all unknowns by mathematical functions. In general, this constructed solution will not satisfy the governing equations (continuity and momentum) because of the arbitrary nature of the choice. But by adding an appropriate source term, which removes any imbalance caused by the choice of the continuum solution, the governing equations are forced to become a model for the constructed solution.

There are 3 kinds of numerical error in unsteady flow calculations: discretization errors, iterative errors and round-off errors. In principle, the main contribution to the numerical error is given by the discretization error, which is caused by the approximation of the time and spatial derivatives. In smooth flow fields computed with 15 digits precision, the round-off error is negligible. Because the Navier-Stokes equations are not linear, the iterative error introduced by the solution of the momentum equations at each time step may also be important. It is unusual in practical unsteady flow calculations to reduce the iterative error to machine accuracy at each time step.

In the MMS framework, it is possible to isolate the 2 main sources of the numerical error, because the exact solution, ϕ_{exact}, is known. For a given time step and grid refinement level, the solution of the momentum equations at each time step is calculated with 2 convergence criteria: the solution converged to machine accuracy, ϕ_o; the solution obtained with criteria typically applied in practical calculations, ϕ_i. The difference $\phi_o - \phi_{\text{exact}}$ gives an excellent estimate of the discretization error, because the iterative error is negligible in ϕ_o. On the other hand, ϕ_o and ϕ_i must have the same discretization error, hence their difference measures the iterative error of the solution ϕ_i. Obviously, the exact error of ϕ_i, $\phi_i - \phi_{\text{exact}}$, is also available, allowing its value to be compared with the contributions of the discretization and iterative errors.

In this paper, we present 2 manufactured solutions (MS) for 2-dimensional, unsteady, laminar, incompressible flows: a periodic solution, and one that evolves to a steady state. A first example of Code Verification with these manufactured solutions is presented for the time-dependent, 2-D, finite-difference version of PARNASSOS (Hoekstra and Eça, 1998). For the sake of completeness, we also present a brief description of the numerical method. The Appendix gives the derivatives of the flow quantities required to compute the source terms to be added to the momentum equations.

MANUFACTURED SOLUTION

The starting point for the construction of the present manufactured solution is the flow field proposed by Eça et al. (2006) for the verification of Reynolds-Averaged Navier-Stokes solvers

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