International Journal of Offshore and Polar Engineering (ISSN 1053-5381)

Vol. 18, No. 1, March 2008, pp. 43–49

Vibration Analysis of Axisymmetric Multi-layer Liquid-filled Cylindrical Containers Using RD-Finite Element Method

L. V. Hai, K. K. Ang and C. M. Wang
Department of Civil Engineering, National University of Singapore, Singapore

This paper is concerned with the free vibration analyses of axisymmetric multi-layer storage containers filled with liquid. A new finite element method (FEM) based on the relative displacement concept is presented for the analysis. The multi-layer cylindrical tank walls are modeled by shell elements which have been developed based on the concept of relative displacements instead of the conventional degrees of freedom involving rotations. The liquid in the containers is discretised by specially designed quadrilateral fluid elements that are kinematically compatible with the shell elements. Some examples are presented to demonstrate the accuracy and effectiveness of the proposed method for the free vibration analysis of multi-layer storage containers containing liquid.

INTRODUCTION

The dynamic interaction between the shell container and the liquid inside it has been a subject of intense study, more so in recent years due to the transportation of liquefied natural gas (LNG) in specially designed LNG vessels as well as the storage of LNG in floating terminals. Many studies on coupled liquid-container interaction have been carried out. For example, Ang (1980) solved this problem by developing an axisymmetric thin shell element, and the effect of liquid sloshing was considered by using a coupling matrix for the shell and fluid elements. Chiba et al. (1985) analyzed a clamped-free cylindrical shell partially filled with an incompressible, inviscid liquid, and the effects of the initial hoop stresses and surface condition on the natural frequencies were investigated. Subhash and Bhattacharyya (1996) employed the FEM that made use of 2-node thin elastic shell elements and 8-node fluid elements for the coupled vibration analysis. Amabili (2000) used the Rayleigh-Ritz method to analyse the vibration problem of cylindrical shells filled with liquid. Further, Cho et al. (2002) addressed analytical and numerical studies on the free vibration of fluid-structure interaction problems considering the fluid compressibility. However, most of the aforementioned studies adopted the classical thin shell theories for modeling the container. These theories ignore the effect of transverse shear deformation. Reddy (1984) showed that the application of classical thin shell theory to multi-layer shells may result in 30% error when used for estimating the natural frequencies. His study concluded that the classical thin shell theories are highly inadequate when used for modeling thick shells and composite laminated shells. So far, vibration analyses of multi-layer shells have been carried out using different approaches. For example, Jing and Tzeng (1993) proposed a refined shear deformation theory of laminated shells by using an independently assumed transverse shear force field. A layer-wise analysis for the free vibration of thick composite cylindrical shells was presented by Huang and Dasgupta (1995), who approximated the displacement field by finite element interpolation shape functions along the thickness direction. Chang and Chiou (1995) obtained the natural frequencies of clamped-clamped laminated cylindrical shells conveying fluid by using Hamilton's principle. Lam and Wu (1999) analyzed thick, rotating, cross-ply laminated composite cylindrical shells using the 1st-order shear deformation shell theory. The analytical solution considered the initial curvature in the stress-strain relationship. George and Pawel (2004) presented a refined theory for thick spherical shells. The results given in this approach incorporated the effects of transverse shear deformation, initial curvature and radial stress.

In general, the foregoing formulations are too complicated for the analysis of thick laminated composite shells. A recent paper by Ma and Ang (2006) provided a solution to the problem. They introduced the relative displacement (RD) concept, which can simplify the computational effort in multi-layer composite shells because it homogenizes the multi-layer as a single layer. The present model captures the shear flexibility and layer-wise in-plane displacement of the laminated composite plates. Based on Ma and Ang's laminated model, we extend the element to handle a multi-layer cylindrical shell containing liquid. Natural vibration frequencies of cylindrical multi-layer storage containers filled with liquid will be determined to illustrate the method's accuracy.

FINITE ELEMENT FORMULATIONS

Mass and Stiffness Matrices of Multi-layer RD-Shell Element

Fig. 1 shows the coordinate system and the geometry of the cylindrical shell with m different material layers through the thickness direction r. The cylindrical shell is assumed to have a height H, mean radius R, total thickness h, and to contain liquid of height H_f. The base of the tank is also assumed to be rigid. For this multi-layer RD-shell, h_i denotes the coordinate of the top of each layer where a pseudo-node is located, and Δu_i denotes the RD of the top of each layer with respect to u_n, the absolute displacement along the meridional direction of the inner (reference) surface. Fig. 2 displays the definition of fiber orientation of a particular layer where 123 is the principal material coordinate, $r\theta z$ the global coordinate system, and α the fiber angle.

In the following, we extend Ma and Ang's (2006) isotropic plate element formulation to an axisymmetric multi-layer shell element. The displacement functions in the kth layer of

---

Received June 3, 2007; revised manuscript received by the editors December 28, 2007. The original version was submitted directly to the Journal.

KEY WORDS: Liquid sloshing, multi-layer shell, relative displacement, fluid-structure interaction, finite element method.