International Journal of Offshore and Polar Engineering (ISSN 1053-5381)

Vol. 18, No. 1, March 2008, pp. 1–7

Spatial Models for Variability of Significant Wave Height in World Oceans

Anastassia Baxevani
Department of Mathematical Statistics, Chalmers University of Technology, Göteborg University
Göteborg, Sweden

Cédric Borgel
UBS/SABRES, Vannes, France

Igor Rychlik
Department of Mathematical Statistics, Chalmers University of Technology, Göteborg University
Göteborg, Sweden

Significant wave height (H_s) is a measure of the variability of the ocean surface. Benefits from knowing the spatial and temporal characteristics of this field are multiple: It is useful to size offshore structures, to foresee the fatigue of the ship's hull depending on its route and season, to compute probabilities of risks associated with marine operations. In this paper, we describe a method for modeling the H_s in space. The method is based on the Gaussian hypothesis for the logarithms of H_s and consists of estimating the mean and the covariance structure of $\log(H_s)$ using the information provided by the total variation. We then use the estimated parameters of every area in the world to construct maps of the median and the correlation structure. These maps are used to compute the probability of H_s exceeding a predefined level, and the distribution of the length of a storm. The data used are those of the TOPEX-Poseidon satellite.

INTRODUCTION

Significant wave height is defined as 4 times the standard deviation of the ocean surface elevation and has been defined to approximate the wave heights visually estimated by experienced mariners. Estimates for significant wave height can be considered to be a 2-dimensional random field in space that develops over time. Models for such spatio-temporal surfaces are generally complex and incorporate many features from dynamic and stochastic systems. From the perspective of model fitting, the complexity is often constrained by the nature and limitations of the data available. Until recently, most of the available wave data consisted of time series of estimates of significant wave height based on buoy and Shipborne Wave Recorder measurements made at a few locations or along ships' routes. The advent of routine global wave measurements from satellites has made it possible to obtain measurements of spatial variability.

In this paper, we propose a method for constructing local models for estimates of significant wave height based on fitting random field models. The data used are those collected by the TOPEX-Poseidon satellite. For convenience, we will not differentiate between the true H_s values and their estimates as obtained from the satellite, and we will use the same notation for both. A full description of the data and some aspects of its limitations can be found in Baxevani, Rychlik and Wilson (2003, 2005). After we introduce some assumptions that are reasonable for modeling the data, we propose a model that is simple and adequate for applications.

Under the assumption that for each fixed time over a bounded region about 4×4 degrees in size the logarithms of the significant wave height values come from a locally stationary Gaussian random field, we wish to estimate the marginal distribution over space, that is, the mean and covariance function which may depend on time. The covariance function is chosen on the basis that it is simple enough given the limitations of the data, but that at the same time it adequately models certain quantities that are important for applications.

To estimate the parameters of the proposed model, a new methodology has been developed based on the total variation of the field along the satellite tracks (Baxevani, Rychlik and Wilson, 2005). The method is applied to data from all over the world. The results are illustrated through atlases that present the median and the covariance parameters. The derived models are then used to compute such quantities as the probability the maximum exceeds a certain level, or the distribution of the length of a storm, etc.

DATA AND MODEL DESCRIPTION

The data consist of estimates of the significant wave height, H_s, taken from the TOPEX-Poseidon satellite, at discrete locations along 1-dimensional tracks over the oceans. The tracks have 2 orientations; those with the same orientation are roughly parallel. For a detailed description of the data used, see Baxevani, Rychlik and Wilson (2003, 2005).

As suggested by Anderson, Carter and Cotton (2001), Baxevani, Rychlik and Wilson(2005), and others, a Gaussian distribution can be used to fit the logarithms of the values of H_s, which are considered to be partial observations of a random field, $\tilde{X}_t(\mathbf{s})$, where t represents time and $\mathbf{s} = (s_1, s_2)$ represents location in space. For a suitable choice of $D \subset \mathbb{R}^2$, the field $\tilde{X}_t(\mathbf{s})$, $\mathbf{s} \in D$ can additionally be assumed to be stationary and differentiable. Under these assumptions and for each fixed time t, the marginal distribution over space of the field $\tilde{X}_t(\mathbf{s})$ is fitted by estimating its mean and covariance structure.

Received July 12, 2007; revised manuscript received by the editors January 15, 2008. The original version (prior to the final revised manuscript) was presented at the 17th International Offshore and Polar Engineering Conference (ISOPE-2007), Lisbon, July 1–6, 2007.
KEY WORDS: Significant wave height, random surface, satellite data, Gaussian random fields, stationary, isotropy.