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Fast Multipole Method for Wave Diffraction/Radiation Problems and Its Applications to VLFS

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This paper presents an accelerated higher-order boundary element method for wave diffraction/radiation problems and its applications, especially for wave response analysis of VLFS (Very Large Floating Structures). The Fast Multipole Method (FMM) has been implemented on the higher-order boundary element code using an 8-node quadrilateral element. The method utilizes an iterative solver, multipole expansion of Green's function, and a hierarchical algorithm using a quadrant-tree. For solving a hydroelastic problem efficiently using an iterative solver, a new algorithm has been introduced, where the equations of motions representing plate vibration are solved at each iterative step. The numerical benchmark calculations have shown the efficiency of the method both in the storage requirement of $O(N)$ and computation time of $O(N \log N)$, where N is the number of unknowns for the velocity potential.

INTRODUCTION

The Boundary Element Method (BEM) using the free-surface Green function has been used as a basic tool for solving diffraction/radiation problems of a floating body. However, when the method is applied to a very large floating structure (VLFS) such as a Mega-Float, the large requirement of computer resources with $O(N^2)$ for storage and with either $O(N^2)$ or $O(N^3)$ for CPU time—which depends on the selection of the solver either as an iterative type or LU-factorization type (N: the number of unknowns)—has made the computation of velocity potentials by BEM impractical.

Hence the computation for a VLFS has been tackled by the finite element method (FEM), where band storage characteristics of the system matrix are utilized, by semi-analytical approaches, or by the BEM utilizing higher-order panels such as B-spline functions (see e.g., Ohtsubo and Sumi, 2000). However, the higher-order panel method based on B-spline functions is hardly convergent for iterative solvers. This paper employs an alternative approach using the Fast Multipole Method (FMM) (Rokhlin, 1985; Barnes and Hut, 1986; Greengard and Rokhlin, 1987; Greengard, 1988; Fukui and Katsumoto, 1997, 1998). The FMM is applied here to accelerate the calculation of the Higher Order Boundary Element Method (HOBEM) using 2nd-order 8-noded quadrilateral panels. The essential theoretical background of the FMM is based on Graf's addition theorem of Bessel functions which appear in the free-surface Green function, the introduction of a hierarchical algorithm to compute the system equations, and the employment of the iterative solver. Although a similar approach to accelerating the BEM for wave diffraction/radiation problems can be found using the precorrected FFT algorithm (Korsmeyer et al., 1996, 1999; Kring et al., 2000), it is limited up to now to a low-order panel and a deep-water case.

Formulations are presented with several benchmark calculations, including wave response analysis of VLFS. The storage requirement of $O(N)$ and CPU time characteristics of $O(N \log N)$ have been confirmed by the benchmark calculations.

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FORMULATIONS

Boundary Value Problem

As shown in Fig. 1, consider a box-like VLFS of the length $2a$ ($=L$), the width $2b$ ($=B$), the draft d, and floating in the open sea of variable depth (with constant depth h at infinity). The variable depth sea-bottom surface (S_B) is defined such that the boundary line (Γ_B) touches on the flat-bottom base-surface of $z=-h$ and the S_B must locate higher than the base surface ($z=-h$). The coordinate system is defined such that the xy plane locates on the undisturbed free surface and the z axis points upward. The center of the VLFS is on the z axis (Fig. 1).

Consider the long-crested harmonic wave with small amplitude. The amplitude of the incident wave is defined by A, the circular frequency by ω, and the angle of incidence by β. ($\beta=0$ corresponds to the head wave from the positive x direction and $\beta=\pi/2$ to the beam wave from the positive y direction.)

Assuming the water to be perfect fluid with no viscosity and incompressible, and the fluid motion to be irrotational, then the fluid motion can be represented by a velocity potential Φ. Also,

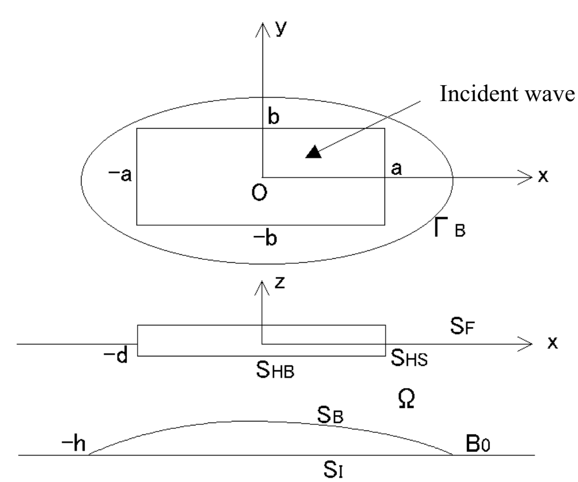

Fig. 1 Configuration of analytical model