

Fundamental Wavenumbers of Diffraction-radiation by a Ship That Advances Through Regular Waves in Finite Water Depth

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A ship that advances at speed U through regular (time-harmonic) waves of frequency ω generates several distinct systems of waves. These wave systems are associated with the separate dispersion curves that are defined by the dispersion relation for ship motions in regular waves. For every value of the parameter $\tau = U\omega/g$ (with $g =$ acceleration of gravity), i.e. in both the subcritical flow regime $\tau < \tau_c$ and the supercritical regime $\tau_c < \tau$, 3 distinct dispersion curves are defined. These separate dispersion curves are fully determined (via parametric representations) in terms of 5 fundamental wavenumbers, which bound 5 distinct ranges of variation of the wavenumber. We consider the critical value τ_c of τ at which a change in flow regime occurs, and the 5 fundamental wavenumbers that bound the separate ranges of variation of the wavenumber and define the related dispersion curves. The results given in the study show that the 3-dimensional theory of ship motions in regular waves can be extended to finite water depth in a straightforward manner.

INTRODUCTION

Diffraction-radiation by a ship that advances at constant speed U through regular (time-harmonic) waves, of frequency ω , in uniform finite water depth D is considered. The Froude number F , the nondimensional wave frequency f , water depth d and wavenumber k , and the parameter τ are defined as:

$$\left. \begin{aligned} F &= U/\sqrt{gL} & f &= \omega\sqrt{L/g} & d &= D/L \\ k &= KL & \tau &= fF = U\omega/g \end{aligned} \right\} \quad (1)$$

Here, L stands for a reference length, e.g. the ship length, and g is the acceleration of gravity. Alternative forms of the nondimensional water depth $d = D/L$ and wavenumber $k = KL$ are:

$$\left. \begin{aligned} d^\omega &= f^2d = D\omega^2/g & k^\omega &= k/f^2 = Kg/\omega^2 \\ d^U &= d/F^2 = Dg/U^2 & k^U &= F^2k = KU^2/g \\ d^{U\omega} &= df/F = D\omega/U & k^{U\omega} &= kF/f = KU/\omega \end{aligned} \right\} \quad (2)$$

These alternative nondimensional water depths and wavenumbers are best suited for particular systems of waves generated by a ship (Noblesse, 2001; Noblesse and Yang, 2004; further on in this study). The relation $\tau = fF$ yields:

$$d^\omega/\tau = d^{U\omega} = \tau d^U \quad \tau k^\omega = k^{U\omega} = k^U/\tau \quad (3)$$

The dispersion relation for diffraction-radiation by a ship advancing through regular waves in finite water depth is:

$$\left. \begin{aligned} \Delta(\alpha, \beta; f, F, d) &= 0 \\ \text{with } \Delta &\equiv k \tanh(dk) - (f - F\alpha)^2 \\ \text{and } k &\equiv \sqrt{\alpha^2 + \beta^2} \end{aligned} \right\} \quad (4)$$

This is well known (e.g. Wehausen and Laitone, 1960; Noblesse and Yang, 2007) and easily verified. Here, α and β are Fourier variables associated with a Fourier superposition of elementary waves. The hyperbolic function $\tanh(dk)$ in the dispersion relation (Eq. 4) can be approximated as:

$$\left. \begin{aligned} \tanh(dk) &\sim 1 \text{ as } d \rightarrow \infty \\ \tanh(dk) &\sim dk - d^3k^3/3 + (2/15)d^5k^5 \\ &\quad - (17/315)d^7k^7 \text{ as } d \rightarrow 0 \end{aligned} \right\} \quad (5)$$

These deep-water and shallow-water approximations are used further on.

The dispersion relation (Eq. 4) defines curves, called dispersion curves, in the Fourier plane (α, β) . These dispersion curves are of fundamental importance. In particular, farfield wave patterns are closely related to, indeed can be directly constructed from, dispersion curves (e.g. Noblesse and Yang, 2007; Chen, 2006). Dispersion curves are also a major ingredient in the construction of Green functions (Noblesse and Yang, 2004; Noblesse and Chen, 1995; Noblesse, Chen and Yang, 1999). The dispersion relation (Eq. 4) yields:

$$\left. \begin{aligned} \alpha &= f/F + (\text{sign } \Delta_f)\sqrt{(k/F^2)\tanh(dk)} \\ \beta &= \pm\sqrt{k^2 - \alpha^2} \end{aligned} \right\} \quad (6)$$

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