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A Study on Mathieu-type Instability of Conventional Spar Platform in Regular Waves

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In this paper, we consider the vertical motion stability of a spar platform based on a damped Mathieu equation for pitch, which is a coupled effect with heave response. Free-decay tests are performed in a wave tank with a scaled model to determine the natural periods and damping coefficients, and the motions in regular waves are measured. During the experiments, it is observed that pitch motions become unstable when the pitch natural period is double the heave natural period. The underlying mechanism is due to the large heave motion making the *GM* negative, and consequently the platform motion becomes unstable. In the process, kinetic energy is transferred from heave mode to pitch mode due to nonlinearity. The experimental results are seen to agree well with the numerical analysis. From the stability diagram, it can be seen that the damping plate and strakes significantly reduce the unstable region compared to the spar platform without any damping devices. The damping devices cannot keep the spar platform from a commensurable relation between periods of heave and pitch, but they are capable of stabilizing motion by increasing the damping and restricting heave resonant motion.

INTRODUCTION

Spar technology has been utilized for offshore structures such as research vessels, communication relay stations, and storage and offloading platforms, and it has extended to deep-draft cylindrical spars for deep-water production (Pauling, 1991; Halkyard, 1996). The conventional spar platform is long and cylindrical, whose large diameter ranges from 30 m to 40 m and is hollow in the middle. Its draft is about 200 m. Due to the deep draft and relatively smaller waterplane area, the structure is insensitive to wave excitations and has long natural periods of motion. For example, the natural periods of the spars deployed in the Gulf of Mexico are 160 s for surge, 60 s for pitch and 28 s for heave. The structure normally exhibits excellent motion characteristics even in extreme sea states. The spar is then regarded as an attractive design solution for regions of ultra deepwater where the water conditions are relatively harsh.

In general, spar platforms show excellent motion behavior in waves, but this is not the case when subject to the action of long swells. The swell period can be very long—up to 30 s, as experienced in the Gulf of Mexico and West Africa. In such a circumstance, the heave motion of spars can be largely amplified.

It was discovered (Adee, 1970) that a long circular cylinder has a large heave motion near its natural period due to small damping. Further, it has been confirmed by scaled model tests that the spar platform undergoes large heave motions at resonance, where its amplitude is enlarged, up to 8 to 10 times higher than the incident wave amplitude (Rho and Choi, 2002). To reduce the large heave motion, various damping devices were introduced, such as a damping plate. We found that the heave plate is effective for reducing the heave resonant motion by 50% (Rho and Choi, 2002).

However, in the case of a spar without a damping device, the large heave motion could induce a coupled pitch motion (Haslum, 2000). The coupling effect between the heave and pitch motions becomes particularly significant when the heave and pitch natural frequencies are commensurable or nearly commensurable ($2\omega_5 \cong \omega_3$). Then, the so-called internal resonance between 2 modes of the system occurs. In addition, if the excitation amplitude is larger than the critical value, very strong nonlinear interaction between 2 modes occurs (Nayfeh, 1979). When the excitation of the 1st-mode frequency ($\Omega = \omega_3$) exceeds a certain limit, not only the 1st mode but the 2nd mode as well become excited. We found from our study that nonlinear behaviors such as saturation and jump can take place.

As mentioned, the heave and pitch natural frequencies of spars in the Gulf of Mexico are about 0.21 and 0.10 rad/s, respectively, and the heave motion is significantly amplified at resonance. Thus it is highly probable that such a nonlinear motion occurs because the nonlinear coupled motion can be evoked. Also, the existence of large pitch motions was observed in model tests by Haslum (2000). He explained the nonlinear phenomena of a circular cylinder with commensurable frequencies through simplified numerical calculations.

In this work, the stability of a spar platform near the heave resonance was investigated both mathematically and experimentally, followed by the examination of the effects of damping plates and strakes. In the mathematical analysis, the equation of pitch motion coupled with heave was derived in the form of a damped Mathieu equation, and the stability diagram of the pitch equation was obtained based on the equation. Free-decay tests were then carried out to validate the numerical results, and to evaluate the natural periods and damping coefficients of heave and pitch motions.

MATHIEU EQUATION

When the spar platform undergoes large heave resonated motions, amplified vertical motion could influence the pitch restoring moment. To investigate the problem in waves, the equation of pitch motion coupled with heave was derived.

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