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Statistical Properties of Wave Groups in Nonlinear Random Waves of Finite Bandwidth

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The theoretical treatment of the nonlinear effect on the formation of a wave group in a random wave field of finite bandwidth is an overdue task hampered by the complicated form of nonlinear random waves. In this study, the mean number of waves in a group, the mean number of waves in a high run and the phase distribution in nonlinear random waves of finite bandwidth are derived. The study employs the complex envelope and total phase function, random variable transformation technique and perturbation method. It turns out that the phase distribution is modified significantly by nonlinearities, showing a systematic excess of values near the mean phase and the corresponding symmetrical deficiency on both sides away from the mean. In the case of the threshold crossing rate, it turns out that it reaches its maximum just below the mean water level rather than at zero. A considerable amount of the threshold crossing rate is shifted toward the larger values of water-surface elevation as nonlinearity increases markedly. Further, the mean waves in a high run associated with highly nonlinear waves are shown to have larger values than their weakly nonlinear counterparts when the reference level is relatively small. A similar trend can also be found in the average number of waves in a group.

INTRODUCTION

The grouping of high waves is an important parameter in many engineering problems associated with port development, which may influence long-period oscillation of moored vessels and other floating structures as well as surf beat. After the Gaussian model was first developed in a well-known paper by Rice (1944), particular attention was paid to the properties of wave groups in Gaussian noise by Longuet-Higgins (1957). Recent interest in this subject (Goda, 1983; Kit et al., 2000) has been stimulated by the suggestion that exceptional damage to ships, coastal defenses or offshore structures may be caused by the occurrence of a run of successive high waves. A further reason for interest is the relation of wave groups to the formation of wave breaking.

Longuet-Higgins (1984) obtained the expressions for wave group length and high run length using 2 apparently distinct approaches in the context of Gaussian waves: First, by a wave envelope function, and later by treating the sequence of wave heights as a Markov chain. It was shown that the 2 approaches are roughly equivalent, and that spectral bandwidth has a significant influence on wave group length and high run length.

On the other hand, the probable effect of nonlinearity on the formation of a wave group in a random wave field of finite bandwidth remains uncertain, due to the complicated form of nonlinear random waves that was first developed by Longuet-Higgins in 1963. In a case where the underlying frequency spectrum is narrow-banded and unidirectional, the stochastic representation of nonlinear sea surface is reduced to a familiar form in which each realization is an amplitude-modulated 2nd-order Stokes wave (Tayfun, 1980, 1986). In contrast to the intricate complexity of the expression of nonlinear waves of finite bandwidth, such an approximation constitutes a simpler formulation with which to study the nonlinear effects on the statistical description of wave properties numerically or analytically. In mild seas, narrow-band assumption has been shown to be reasonably accurate. However, in high seas, this assumption may not be satisfactory.

Later Tung et al. (1989) carried out the search for a way simpler than Longuet-Higgins's (1963) to describe nonlinear waves of finite bandwidth. Based on Tayfun's studies (1980, 1986), Tung et al. (1989) proposed a simple but accurate expression for 2nd-order nonlinear wave elevation for waves of moderate bandwidth.

Cho and Yoon elaborated on this wave model (1992) to analyze the extreme distributions of wave elevation. It turns out that as nonlinearity grows more profound, these extreme distributions deviate from the linear counterpart. The general character of this deviation is in the form of a spreading of the density mass toward the larger and smaller values of crest. Hence, it can be deduced from this tendency that there should be a significant modification in wave group properties when nonlinearity is introduced.

The objective here is to gain some theoretical insight into the nature of nonlinear effects on the statistical properties of a wave group, in terms of wave group length and high run length, quantities that are of great importance in the design of ships, coastal defenses or offshore structures. In this paper, our attention is centered on deep-water waves only.

REVIEW OF WAVE GROUP THEORY

It is known that for the stationary random process of arbitrary bandwidth $\varsigma(t)$, the average number of waves in a group G and the mean number of waves in a high run H (Lin, 1967) are:

$$G = N_\varsigma(\varsigma_0)/N_A(A_0) \tag{1}$$

$$H = N_\varsigma(\varsigma_0)Q(A_0)/N_A(A_0) \tag{2}$$

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