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Comparison of Classical and Simple Free-surface Green Functions

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ABSTRACT

The classical Green function and a simpler Green function associated with the linearized free-surface boundary condition for diffraction-radiation by a ship advancing through regular waves are compared in the special case of steady flows. Both the classical Green function and the simpler Green function satisfy the radiation condition and the linearized free-surface boundary condition in the farfield, where the linear free-surface condition is valid. The classical Green function also satisfies the linear free-surface condition in the nearfield, where this linear condition is only an approximation, while the simpler Green function satisfies the linear free-surface condition only approximately in the nearfield. Numerical differences between these 2 alternative free-surface Green functions vanish in the farfield as expected, and they are relatively small in the nearfield.

INTRODUCTION

Diffraction-radiation by an offshore structure in waves or a ship advancing at constant speed in waves, within the context of a potential-flow frequency-domain analysis, is a classical problem that has been extensively considered in the literature. For offshore structures, robust and practical panel methods have been developed and are routinely used in the offshore industry, to solve the canonical wave diffraction-radiation problems that yield added-mass and wave-damping coefficients, and wave exciting forces and moments. These panel methods solve a boundary-integral equation based on the Green function that satisfies the linear free-surface boundary condition for diffraction-radiation of time-harmonic waves without forward speed.

Application of this classical approach (often identified as the free-surface Green function method in the literature) to wave diffraction-radiation by ships (i.e. with forward speed) has also led to useful methods (Diebold, 2003; Boin et al., 2002, 2000; Chen et al., 2000; Guilbaud et al., 2000; Fang, 2000; Wang et al., 1999; Du et al., 2000, 1999; Iwashita and Ito, 1998; Iwashita, 1997), although not to a comparable degree of practicality because forward speed introduces major difficulties.

A basic difficulty is related to the property that the boundary-integral representation of the velocity potential for time-harmonic (or steady) free-surface flows about a ship advancing with speed $\mathcal{U}$ in waves (or in calm water) involves a surface integral over the ship hull and a line integral around the ship waterline, as first shown in Brard (1972) for steady flows. This line integral (not present if $\mathcal{U}=0$) has important effects. In particular, the waterline integral is shown in Ba et al. (2001) to have a determining influence on irregular frequencies. Further, the contributions of the waterline integral and of the hull-surface integral largely cancel out (Noblesse et al., 1990; Noblesse and Yang, 1995; Yang et al., 2004), which can result in a loss of accuracy.

Another basic difficulty is that the Green function that satisfies the linear free-surface boundary condition for diffraction-radiation of time-harmonic waves (frequency ω) with forward speed $\mathcal{U}$ is considerably more complicated than the Green functions corresponding to the special cases $\mathcal{U}=0$ or $\omega=0$, which can be evaluated relatively simply and efficiently, at least in deep water (e.g., Ponizy et al., 1994). Several free-surface Green functions, based on alternative mathematical representations, have been proposed and used in the literature on wave diffraction-radiation with forward speed. Briefly, 2 main types of free-surface Green functions have been used: 1. Green functions defined by single Fourier integrals that involve relatively complicated special functions (related to the complex exponential integral) of a complex argument; and 2. Green functions expressed as single Fourier integrals along a steepest-descent integration-path (that must be determined numerically) in the complex Fourier plane (Bessho's method). These free-surface Green functions, and related singularity distributions over flat rectangular or triangular panels, have been considered in numerous studies, and relatively efficient numerical-evaluation methods have been developed (Maury, 2000; Chen, 1999; Boin et al., 1999; Brument and Delhommeau, 1997; Ba and Guilbaud, 1995; Iwashita and Ohkusu, 1992; Bougis and Coudray, 1991; Jankowski, 1990; Hoff, 1990; Wu and Eatock Taylor, 1987; Guevel and Bougis, 1982; Inglis and Price, 1982; Kobayashi, 1981; Bessho, 1977; Wehausen and Laitone, 1960). Nevertheless, Green functions that satisfy the free-surface boundary condition for wave diffraction-radiation with forward speed are relatively complicated building blocks.

These 2 basic difficulties of a frequency-domain analysis of wave diffraction-radiation with forward speed are examined in Noblesse and Yang (2004a) and Noblesse and Yang (2004b), where an alternative free-surface Green function and an alternative boundary-integral representation of the potential are given, respectively. The classical Green boundary-integral representation of the potential and the alternative weakly-singular potential representation given in Noblesse and Yang (2004b) are compared in Yang et

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KEY WORDS: Diffraction-radiation, Green function, frequency domain, steady flow, free-surface flow.