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The Scattering of Flexural-Gravity Waves by an Ice Field

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ABSTRACT

A new model is reported that describes the passage of ocean waves in a sea ice medium that varies spatially (Williams and Squire, in the press). Ridges of arbitrary shape, open and refrozen leads, and cracks can be treated, either singly, in groups, or as random distributions. Different orientations can also be introduced. Two approximations are also presented to facilitate computational efficiency and speed. In this paper the model is used to investigate how the various irregularities present in a typical ice field influence the passage of flexural-gravity waves, and which types of feature cause the observed low-pass filter response.

INTRODUCTION

The bulk of published theoretical work describing how ocean waves travel as flexural-gravity waves in a sea ice medium assumes that the ice sheet is homogeneous. This is actually far from the case, as sea ice is peppered with imperfections. At the microscale its morphology is determined by its growth history, including both thermodynamical and dynamical effects, and this influences its mechanical properties. At larger scales, its appearance is dominated by pressure ridges that may be consolidated or loose aggregates of broken ice pieces, open and refrozen leads and polynyas, uneven snow loading, etc. The basis for assuming uniformity, namely that waves are generally long in comparison with the imperfections, is plausible so long as the information desired from the theory is undemanding, or the waves' fetch is not too large. When details about the ice cover, such as mean ice thickness or aspects of the physical properties are sought, however, the model ice sheet has to include a parameterization of the heterogeneity, and the solution technique must not preclude spatial variation.

There has recently been some success in modelling abrupt changes of property, e.g. changes in thickness (Squire and Dixon, 2001a) and propagation across one or more open cracks (Squire and Dixon, 2000, 2001b; Williams and Squire, 2002; Evans and Porter, 2003). Barrett and Squire (1996) investigated the problem numerically, using an extension of the matching method of Fox and Squire (1994). Marchenko (1997) also models a crack and a pressure ridge, the latter as a point linear irregularity that permits a solution to be found in much the same way as for a single crack.

The approach of this paper is rather different. Here a region of sea ice, bordered on either side by ice sheets with uniform properties and thickness, varies spatially in some prescribed way over a finite width. Within the inhomogeneous zone, the thickness can change or any of the material properties may vary. The zone may be separated from the encircling uniform sheet by a crack, or it may be frozen to it, allowing many different classes of irregularity to be considered, including the open lead. The model is constructed to be as general as possible, because of the many configurations of heterogeneity that are possible in sea ice—depending on its growth and deformation history. Multiple features can be examined, either by treating the irregularities as part of a continuous variation across the inhomogeneous zone, or approximately by dealing with each separately and computing their effect in sequence—the No Evanescent Waves (NEW) and serial approximations. It is found that the poorest approximate solution traces the progression of the exact solution well, but without the presence of the fine structure that arises due to interference between identical features.

This paper considers cracks, open and refrozen leads, and some example ridge configurations, extending the theory and results presented in Williams and Squire (in the press). To better represent a real ice field, such as might occur in the Arctic Basin, random distributions of irregularities are also considered, including a mixture of different features computed serially.

FORMULATION

In constructing a model for flexural-gravity wave propagation in an ice sheet, we are mindful of the considerable literature suggesting that the behaviour of the sheet may be represented as a thin elastic plate at the strains and strain rates involved. While some damping of the waves undoubtedly occurs—indeed, viscoelastic plates have occasionally been employed—the additional complication of including viscosity is unwarranted here. We are helped considerably by the observation that the wavelength of ice-coupled, flexural-gravity oscillations is always much greater than the ice thickness or the submergence, even when the wave period is quite short. Accordingly, we are at ease that sea ice can be described by a Euler-Bernoulli thin plate floating with zero submergence on the water surface at $z = 0$.

This paper deals with 3 types of irregularities: pressure ridges, leads—which may be either open or refrozen—and open cracks. However, each feature can be approximately described by the same general equations, and these are presented here.

Equations and Boundary Conditions

Fig. 1 shows the general situation to be modelled and the coordinate system used. The main ice cover is of constant thickness h_0, and the seawater has a finite depth of H. The coordinate axes are oriented as shown, but are displaced to the right in the

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