

Application of the p-Version FEM to Stress Singularity Field Problems

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ABSTRACT

Two types of p-version finite elements were developed, which enable precise analyses of hot spot stress at a welded zone and of the stress intensity factor at a root gap by fairly coarse meshing. The stress singularity exists at the crack tip and the weld toe. Stress singularities are expressed by the term of $-1/2$ power of distance r from the singular point for the crack tip, and of $-1/3$ power for the weld toe. Displacement is expressed by $1/2$ power and $2/3$ power of r for the respective cases. For those displacement expressions, 2 types of polynomial function elements were developed. Their transformations from natural coordinates to actual coordinates are biased toward the singular points. To verify the performance of the elements, 2 examples were demonstrated. The first is a 2-edged long plate with a perpendicular crack on one edge, under uniaxial uniform tension at the far ends. The stress intensity factor was calculated by just 2 elements through the plate width, and compared with the conventional reference. The second example is a T-joint with fillet welding under uniaxial uniform tension at the 2 far ends of the flat plate. Hot spot stress at the welded zone was calculated with only 2 elements through the thickness. The result was compared with that by BEM with a very fine mesh. Good agreement between the results obtained by the elements and reference solutions showed that the elements could provide high precision for the welded zone. This fact implies that the newly developed polynomial elements could reduce meshing labor.

INTRODUCTION

For welded structures such as ships and rolling stock, a technique of precise evaluation of the fatigue strength of the welded joints is a critical factor for assuring product quality. The present general method for fatigue strength evaluation depends primarily on the classification of the joints and the corresponding allowable stresses (Miki, 1993). This complicated method could cause the following ambiguity and complexity:

- The allowable stress, which is defined as nominal stress in the classification, is hard to be defined on the actual structures.
- Application to the classification needs specialized knowledge on fatigue strength and structural mechanics.

The evaluation method that uses hot spot stress (HSS) (Nihei, 1993) has been suggested as a method for resolving these difficulties. This method necessitates precise HSS by costly and laborious numerical methods, using fine FE meshing on the vicinity of the weld toe by solid elements. The complexity of a stress intensity factor (SIF) analysis, which is required for evaluating the fracture from the root gap, is similar to an HSS analysis.

Although a possible means for reducing the labor by eliminating the need for fine mesh division is the use of the p-version FEM, which allows a highly precise analysis through rough mesh division, it is not thought to be suitable for analyses of cracks and weld toes where stress singularities exist (Yuuki, 1992).

As the countermeasure for drawbacks in the conventional p-version finite element, 2 types of p-version finite elements were developed. These elements make it possible to perform an HSS analysis and an SIF analysis with high accuracy. This paper describes the basic theory and gives examples of the application.

BASIC THEORY OF P-VERSION CRACK ELEMENT

Displacement u and stress σ in the vicinity of a crack are expressed by Eqs. 1 and 2 as functions of the distance x from the crack tip:

$$u = a_1 + a_2x^{\frac{1}{2}} + a_3x + a_4x^{\frac{3}{2}} + a_5x^2 + a_6x^{\frac{5}{2}} + \dots \quad (1)$$

$$\sigma = b_1x^{-\frac{1}{2}} + b_2 + b_3x^{\frac{1}{2}} + b_4x + b_5x^{\frac{3}{2}} + \dots \quad (2)$$

Using a nonlinear mapping between natural and actual coordinates, the crack element, which can describe Eqs. 1 and 2, was developed.

Displacement Functions of Conventional p-Version Element

In the p-version finite element code (Kawasaki, 1997), displacement of the corner points and amplitudes of the deformation modes of the sides, faces and a body that comprise the element are handled as unknown quantities. The displacement functions of the sides, faces and a body were derived from Legendre's series.

As an example of formulation, the displacement functions of a 1-dimensional element are shown. Although the p-version FE code allows handling up to the 9th order of displacement functions, this paper only shows up to the 5th order (Fig. 1).

1. Displacement functions for corner points:

$$N_1 = \frac{1}{2}(1 - \xi) \quad N_2 = \frac{1}{2}(1 + \xi) \quad (3)$$

2. Displacement functions for sides ($N_i^{(l)}$ = displacement function of i -th order):

$$\begin{aligned} N_2^{(l)} &= 1 - \xi^2 \\ N_3^{(l)} &= \frac{3\sqrt{3}}{2}\xi(1 - \xi^2) \\ N_4^{(l)} &= (\xi^2 - 1)(5\xi^2 - 1) \\ N_5^{(l)} &= \alpha\xi(\xi^2 - 1)(7\xi^2 - 3) \quad \alpha = 1.518 \end{aligned} \quad (4)$$

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