International Journal of Offshore and Polar Engineering
Vol. 13, No. 2, June 2003 (ISSN 1053-5381)

Statistical Properties of Nonlinear Froude-Krylov Forces on Cylinders

Felice Arena* and Francesco Fedele
Department of Mechanics and Materials,
Mediterranea University of Reggio Calabria, Reggio Calabria, Italy

ABSTRACT

The statistical properties of the second-order Froude-Krylov force on a cylinder (whether a vertical cylinder or a horizontal submerged cylinder), for narrow-band spectra, are investigated. Two families of stochastic processes are defined for this purpose, and for each family the probability density function and the probabilities of exceedance of the absolute maximum and of the absolute minimum are obtained. It is then proven that the above-mentioned Froude-Krylov force processes belong to these stochastic families. The predictions for the Froude-Krylov force on a horizontal submerged cylinder agree with the results of a small-scale field experiment.

INTRODUCTION

The amplitude of the wave force on a large structure may be obtained as the product of the Froude-Krylov force (defined as the force on the equivalent water volume) and the diffraction coefficient of the wave force (Sarpkaya and Isaacson, 1981). Thus it is helpful for the design of large offshore structures to investigate the properties of the Froude-Krylov force.

According to the linear theory of wind-generated waves (Longuet-Higgins, 1963; Phillips, 1967), the linear Froude-Krylov force, whether on a vertical cylinder or on a horizontal submerged cylinder, represents a random Gaussian process of time. Thus both the absolute maximum and the absolute minimum of the linear Froude-Krylov force have the same Rayleigh distribution, if the spectrum is very narrow (Longuet-Higgins, 1952).

Boccotti (2000) has shown that, for large horizontal cylinders, the 2 random processes—the wave force on the solid cylinder and the Froude-Krylov wave force—have nearly the same very narrow spectrum, the same nonlinearity effects, and the same statistical properties: equal distribution of the normalized crest-to-trough heights, distribution of the normalized absolute maximum and distribution of the normalized absolute minimum. This conclusion is based on the evidence of a small-scale field experiment which consisted in the real-time comparison of the wave forces on a horizontal submerged cylinder and on an ideal equivalent water cylinder. (Also see Boccotti, 1996, and Arena, 2002.)

The statistics of nonlinear wave forces were studied by Naess and Johnsen (1992), who proposed a numerical approach for calculating the probability density function of the second-order hydrodynamic loads and the response of compliant offshore structures.

In this paper, an analytical formulation is proposed for narrow-band, wind-generated wave processes (Tayfun, 1980), including the nonlinear Froude-Krylov forces on cylinders. In detail we define 2 families of nonlinear stochastic processes, ψ_1 and ψ_2, the first consisting of statistically symmetric processes, the second of statistically nonsymmetric processes. For each family of random processes we obtain the probability density function, the probability of exceedance of the absolute maximum and the probability of exceedance of the absolute minimum. For the family ψ_1, these properties depend upon one parameter, δ; for the family ψ_2, they depend upon 2 parameters, α_1 and α_2.

We prove that the horizontal component of the narrow-band second-order Froude-Krylov force (whether on a vertical cylinder or on a horizontal submerged cylinder) represents a random process of time which belongs to the stochastic family ψ_1, and the expression of parameter δ is derived for this process. We prove also that the vertical component of the narrow-band second-order Froude-Krylov force (on a horizontal submerged cylinder) represents a random process of time which belongs to the stochastic family ψ_2, and the expressions of parameters α_1 and α_2 are obtained for this process.

Finally, we show that, based on experimental evidence, the analytical predictions for the Froude-Krylov force on a horizontal submerged cylinder agree with the conclusions of Boccotti (2000).

STATISTICAL PROPERTIES OF 2 STOCHASTIC FAMILIES WITH NARROW-BAND SPECTRUM

Let us define the 2 families of stochastic processes of time:

$$\psi_1(t) = f_1 a \sin[\chi(t)] + g_1 a^2 \sin[2\chi(t)] \tag{1}$$

$$\psi_2(t) = f_2 a \cos[\chi(t)] + g_2 a^2 \cos^2[\chi(t)] + h_2 a^2 \sin^2[\chi(t)] \tag{2}$$

where a is a Rayleigh distributed random variable; f_1, g_1, f_2, g_2 and h_2 are parameters with some fixed values; and $\chi(t) = wt + \varphi$, with w the angular frequency, and φ a random phase uniformly distributed in $(0, 2\pi)$.

Probability Density Functions of Stochastic Family ψ_1

Let us consider the normalized random process:

$$\zeta_1 = (\psi_1 - \bar{\psi}_1)/\sigma_{\psi_1} \tag{3}$$

where $\bar{\psi}_1$ and σ_{ψ_1} are, respectively, the mean value and the standard deviation of random process ψ_1. Defining the 2 Gaussian

*ISOPE Member.

Received May 13, 2002; revised manuscript received by the editors February 17, 2003. The original version (prior to the final revised manuscript) was presented at the 11th International Offshore and Polar Engineering Conference (ISOPE-2001), Stavanger, Norway, June 17–22, 2001.
KEY WORDS: Wind-generated waves, Froude-Krylov force, nonlinearity effects, probability of exceedance of absolute maximum, probability of exceedance of absolute minimum, vertical cylinder, horizontal submerged cylinder.