

## Artificial Damping in Water Wave Problems II: Application to Wave Absorption

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### ABSTRACT

This paper presents wave absorption using artificial damping, aiming the application at the radiation condition of water wave problems. The dispersion relation in the wave absorbing area is derived, and a series of numerical tests for different damping parameters is introduced. It is found that the extra term used to control the dispersion relation plays a critical role, and that a quadratic function is desirable for the variation of damping strength.

### INTRODUCTION

Use of the radiation condition for water wave problems in an unbounded domain yields a unique and energy-conserving solution. For an accurate and stable numerical simulation of wave propagation, a proper treatment of the radiation condition is essential. The waves reflecting from a truncated boundary should be minimized, and wave direction should be properly controlled.

Some representative numerical techniques for the radiation condition of water waves are Orlandi's method (e.g. Chan, 1977), matching with linear/far-field solutions (e.g. Bai and Yeung, 1981), an artificial damping zone for wave absorption (e.g. Baker, Meiron and Orszag, 1982), and a periodic condition (Vinje and Brevig, 1981). Of these methods, the one popular today is the artificial damping concept. This method is based on absorbing waves in a certain region, the so-called damping zone or artificial damping beach, by adding a strong damping mechanism.

The absorption of waves for the radiation condition was introduced by Israeli and Orszag (1981) for wave equations, and extended to ship waves by Baker, Meiron and Orszag (1982). Later, Coite (1989) showed its numerical efficiency for a 2-dimensional wave-maker problem, and Clements (1995) successfully combined this technique with a piston-type wave absorber. All of those applications for water wave problems considered only 1 linear damping coefficient for the free-surface boundary condition. Adopting the original idea of Israeli and Orszag, Nakos (1993) included another term to disallow the change of the linear dispersion relation in the damping region, and this method was successfully applied to the ship-motion problem. Kim (1997) showed that this method is valid for second-order wave simulations.

Despite many applications of the artificial damping zone, proper guidance for the optimum damping parameters is not yet clear. The work of Israeli and Orszag included a thorough theoretical background of the damping mechanism, but their study was limited to a few representative wave equations valid for atomic waves. A deep and thorough study on this issue is thus very

valuable for the efficient and effective numerical simulation of water waves in an unbounded domain.

This paper introduces a series of numerical tests for different damping parameters. In the previous study for constant damping ("Artificial Damping in Water Wave Problems I: Constant Damping"), we saw that equivalent linear damping can be designed with and without a change of dispersion relation. We also observed that, although the velocity potential is the same, wave elevation is dependent on the formulation of the kinematic boundary condition. In this study of the radiation condition, it is shown that the efficiency of wave absorption is more significantly dependent than the constant damping case.

For the numerical tests, Rankine panel methods have been applied to 2- and 3-dimensional wave problems. Both linear and fully nonlinear free-surface boundary conditions are considered, and the wave elevation and potential wave energy are observed. In particular, the results are compared for 2 different methods with and without a Newtonian cooling term, showing a very dramatic difference.

### LINEAR WAVES IN VARYING DAMPING DOMAIN

Let's consider the following form of the linearized free-surface boundary condition with 2 additional damping terms:

$$\frac{\partial^2 \Phi}{\partial t^2} + g \frac{\partial \Phi}{\partial t} + \mu_1 \frac{\partial \Phi}{\partial z} + \mu_2 \Phi = 0 \quad (1)$$

where all notations are the same as in our earlier paper. Plus, we will consider the polynomial forms of  $\mu_1$  and  $\mu_2$  such that:

$$\mu_1 = \sum_{m=0}^{M1} \sum_{b=0}^{N1} A_{mb} x^m y^b \quad \text{and} \quad \mu_2 = \sum_{m=0}^{M2} \sum_{b=0}^{N2} B_{mb} x^m y^b \quad (2)$$

Assuming  $\Phi = \phi e^{i\omega t}$ , we can write:

$$\frac{\partial \Phi}{\partial z} = -\frac{1}{g} (-\omega^2 + i\omega\mu_1 + \mu_2) \Phi. \quad (3)$$

Now let's consider Green's second identity for a simple disturbance case such that:

$$2\pi \Phi - \iint_{S_F} \frac{\partial \Phi}{\partial z} G ds = R \quad (4)$$

where  $G$  is the Green function, and the Rankine source is considered in this case, i.e.  $G = 1/\bar{r}$  where  $\bar{r}$  is the distance between a

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