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On the Estimation of Nonlinear Volterra Models in Offshore Engineering

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ABSTRACT

A quadratic Volterra model is used to describe the nonlinear relationship between the sea-waves and a mini-TLP surge drift motion, and the practical problems in estimation of the Volterra model from a limited number of random data is discussed. The main difference between assuming Gaussian and non-Gaussian sea-waves in the Volterra estimation is discussed, and we show that the statistical properties of Volterra model estimation are closely connected to the properties of the higher-order spectral moment estimators applied. The use of a spectral smoother, such as the multitaper approach described in this paper, provides considerably lower variance than the standard periodogram based on higher-order spectral moment estimators. In the non-Gaussian input Volterra model case, where the fourth-order spectral moment of the input has to be estimated, we find that the inversion of the input spectral matrix is very sensitive to noise. Thus, we propose a principal component analysis (PCA) method which yields considerably lower variance in the estimation of Volterra models for small data sets. Of particular interest to offshore engineering, the combination of multitaper estimation and the PCA method provides very good results in a mini-TLP model test, which may lead to possible new insight in the nonlinear interaction between sea-wave excitation and offshore structural response.

INTRODUCTION

Nonlinear Volterra models can be used in offshore engineering to classify and quantify the nonlinear relationship between random sea-waves and the motion of offshore structures (Bendat, 1990; Bar-Avi and Benaroya, 1997). Second-order Volterra models (particularly frequency domain models) have been successfully applied to describe effects such as low-frequency drift motion of moored or tethered structures such as tension leg platforms (TLPs) (Kim et al., 1991; Kim and Powers, 1995; Stansberg, 1997; Stansberg, 2001).

Previously, we have shown that the assumption of Gaussian input when the input in fact is non-Gaussian leads to erroneous Volterra models (Kim and Powers, 1988; Kim and Powers, 1995). We have also shown that the quality of the higher-order (cross-) spectral estimates used in the calculation of the Volterra model plays a significant role in obtaining high-quality model results (Birkelund and Powers, 2001).

In this paper, we describe the difference in Volterra estimation when Gaussian and non-Gaussian inputs are assumed, and point out the pros and cons in the resulting Volterra model estimation method. The statistical properties of Volterra model estimation are in both cases shown to be directly connected to the properties of the chosen spectral moment estimator method. In the non-Gaussian case, the estimation of the input spectral moment up to the fourth order is shown to be a critical factor. On this basis, we suggest appropriate spectral estimators for commonly encountered cases in offshore engineering.

Because of the strong relationship between the quality of the Volterra model results and the variance in the hierarchy of input spectral moments, we also propose an eigenvector decomposition-based method for calculating the Volterra kernel in the non-Gaussian input case. This method eliminates numerical instabilities marring the standard calculations, and thus provides a considerably faster convergence and more accurate results. Such a method is of particular interest in analyzing offshore model basin tests, where long-term data often are limited due to experimental costs. To visualize the superiority of this new method, we have chosen to use data from a mini-TLP model basin test carried out at the Offshore Technology Research Center at The Texas A&M University.

VOLTERRA MODEL

Given the time domain zero mean input data $x(t)$ and output data $y(t)$ with Fourier transforms $X(f)$ and $Y(f)$, respectively, there exist several methods to analyze their nonlinear relationship (Bendat, 1990; Bar-Avi and Benaroya, 1997). The Volterra functional series representation of a quadratical nonlinear system can be written in the frequency domain as (Bendat, 1990):

$$Y(f) = H_L(f)X(f) + \int H_Q(f, f-f')X(f')X(f-f')\,df' + R(f). \qquad (1)$$

Here, $H_L(f)$ and $H_Q(f_1, f_2)$ are referred to as the linear and quadratic transfer functions, respectively, and $R(f)$ is a zero mean model error (due to neglected higher-order terms and/or neglected inputs).

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KEY WORDS: Nonlinear system identification, Volterra model, principal component analysis, higher order spectra, multitaper estimators.