

## Limitation of 2nd-order Theories for Laboratory High Sea Waves and Forces on Structures

Amitabh Kumar and Cheung Hun Kim\*  
Civil Engineering Department, Ocean Engineering Program  
Texas A&M University, College Station, Texas, USA

Jun Zou  
ABB Lummus Global Oil & Gas-America, Houston, Texas, USA

### ABSTRACT

We searched the limitation of the 2nd-order random wave and diffraction theories affected by the sea's severity, in comparison with laboratory data. The 1st-order theory agrees approximately with the waves till the "rough sea state" ( $H_s \leq 4$  m), and the 2nd-order theory approximately agrees with experimental data till the "high sea state" ( $H_s \leq 9$  m). The agreement is better in the lower sea states ( $H_s \leq 6$  m) than the high sea state ( $H_s \leq 9$  m). The theory increasingly underestimates as the severity increases from "high sea" to "phenomenal sea" ( $9 \text{ m} \leq H_s$ ). The 2nd-order diffraction theory is in excellent agreement with the experiment of a structure in the waves of  $H_s = 8.0$  m, which is indeed within the limitation of the 2nd-order random wave theory ( $H_s \leq 9$  m).

### INTRODUCTION

The 2nd-order random wave and 2nd-order diffraction theories have been frequently used in the recent past for the study of nonlinear responses of offshore structures. Although the effect of the other higher-order nonlinear terms might be significant in some cases, the effect of the 2nd-order theories has met with the widespread attention of today's engineers, and they have been helpful in solving some critical offshore design problems. However, little work has been carried out on the limitation of the 2nd-order theories for the waves and forces affected by the sea's severity. The objective of this study is to investigate and attempt to propose the limitation of 2nd-order random wave and diffraction theories in comparison with some limited experimental data.

As the 2nd-order diffraction force on a large offshore structure is due to the 2nd-order waves, the 2nd-order diffraction theory has to agree with the experimental wave force due to the same laboratory sea waves that agreed with the 2nd-order wave theory. Thus it may be redundant to study further the limitation of diffraction theory in depth. However, this paper will discuss the recent comparison with the experimental data relevant to wave forces obtained in the waves of  $H_s = 8.0$  m, which is in fact within the limitation of the 2nd-order random wave theory ( $H_s < 9$  m).

### COMPARISON OF WAVES

#### 2nd-order Random Waves

Theories of the 2nd-order multi-directional sea waves have been put forth by Longuet-Higgins (1963) for deep water and by

Sharma and Dean (1981), Dean and Sharma (1981) and Dalzell (1999) for finite water. We consider the long-crested sea waves for finite to deep water in our comparison with the available wave tank data. The 2nd-order long-crested random wave by Sharma and Dean (1981) is:

$$\eta(x, y, t) = \sum_{i=1}^N a_i \cos(\psi_i) + \frac{1}{4} \sum_{i=1}^N \sum_{j=1}^N a_i a_j \left\{ \begin{array}{l} K_{ij}^+ \cos(\psi_i + \psi_j) \\ + K_{ij}^- \cos(\psi_i - \psi_j) \end{array} \right\} \quad (1)$$

$$\psi_i = k_i x - \omega_i t + \varepsilon_i \quad \omega_i^2 = g k_i \tanh k_i h$$

where  $K_{ij}^{\pm}$  is found in Sharma and Dean (1981), and  $\varepsilon_i$  is the random phase angle uniformly distributed over 0 to  $2\pi$ . Without loss of generality, we may take  $\eta(x, y, t)$  at the origin of the coordinate system. Then Eq. 1 may be put into the form identical to the Volterra quadratic input-output model, to be shown later. In the simulation of the 2nd-order wave, the input is Gaussian wave (1st-order) and the output is the sum of the 1st- and 2nd-order random waves as shown in Eq. 1.  $K_{ij}^{\pm}$  consists of the mean, double frequency, sum and difference frequency terms. Given the target energy density spectrum, one can determine the linear random waves and add the computation of the quadratic order part in Eq. 1 to produce the random waves to the 2nd-order.

#### Simulation and Comparison of 2nd-order Random Waves

In the wave tank, the random waves are generated using the driving signal generated from the target energy density spectrum. The digital simulation of the 2nd-order random waves is also achieved based on the same target energy spectrum. Thus, the target spectrum is the common basis for comparison of the simulation and experimental data. Comparison of these will provide the basic information for a probabilistic judgement for the limitation of the theories. We compute first the wave amplitudes from the target spectrum; then we take  $\omega_{\max}$  as double the Nyquist frequency

\*ISOPE Member.

Received December 26, 2001; revised manuscript received by the editors September 8, 2002. The original version was submitted directly to the Journal.

KEY WORDS: 2nd-order random wave theory, crest height distribution, 2nd-order diffraction theory, Volterra 2nd-order model, TLP tendon tension.