International Journal of Offshore and Polar Engineering
Vol. 12, No. 3, September 2002 (ISSN 1053-5381)

Wave Propagation Across an Oblique Crack in an Ice Sheet

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ABSTRACT

Oblique waves are propagated across a crack in an infinite homogeneous ice sheet that is modelled as an Euler-Bernoulli thin plate. The water beneath is taken to be infinitely deep and of constant density, and the flow is assumed to be inviscid and irrotational. An integral representation of the appropriate Green's function is found, from which are deduced analytical formulae for the reflection and transmission coefficients. The paper extends the work of Squire and Dixon (2000), which is concerned with waves at normal incidence.

INTRODUCTION

Deep into the Arctic Ocean, many hundreds of kilometres from the nearest ice edge, the sea-ice cover is known to exhibit small oscillatory motions that arise due to the presence of ice-coupled waves. These waves, recorded by both tiltmeters and strain gauges, originate as surface gravity waves in the open sea to the south. As they impinge on the ice edge, the surface gravity waves are partially transmitted as ice-coupled waves, in a way that favours the propagation of longer periods at the expense of short-period waves because of the impedance change at the ice margin. Within the heterogeneous ice veneer, the ice-coupled waves encounter many flaws, such as pressure ridges and cracks, which affect their passage. These imperfections also discourage short waves (Squire and Dixon, 2000), as does the hysteresis in the ice sheet itself that results from the inherent inelasticity of sea-ice.

Squire and Dixon (2000) model how ice-coupled waves are affected by an open crack met at normal incidence. While this problem has been studied in the past (Barrett and Squire, 1996), it is shown by Squire and Dixon that simple algebraic formulae emerge for the reflection and transmission coefficients, and that the numerical solution developed in the 1996 paper is unnecessary. The current study takes the analysis a step further, allowing the ice-coupled waves to be incident on the crack at any angle. This removes an intrinsic simplification of the earlier work, namely 2-dimensionality, although the 3-dimension problem solved here is tackled by first transforming it into a 2-dimensional, modified Helmholtz boundary value problem.

While the focus of this paper is entirely on ice-coupled waves travelling beneath an infinite, inhomogeneous ice sheet, an equally appealing application of the work is to waves propagating obliquely across a sequence of flexible pontoons that may serve as breakwaters or an energy conversion device, or to a crack in a floating airport. The former problems can be dealt with by considering a succession of open-water zones between flexible rafts in the manner of Squire and Dixon (2001), while the current solution is directly relevant to the latter problem so long as the open-crack edge conditions are suitable. Since it is unlikely that such an extreme crack would be present on a runway, an analysis with a modified crack parameterization is desirable; however, it is unclear at this time how to represent the crack realistically.

EQUATIONS AND BOUNDARY CONDITIONS

Fig. 1 gives a schematic representation of the physical situation to be modelled, including the coordinate system that will be used. There is an infinite sheet of ice floating at the surface $z = 0$ with an infinitely long crack along the line $x = 0$. A sinusoidal wave approaches the crack at an angle θ to the x-axis.

The flow of the seawater beneath the ice is taken to be irrotational. So, assuming waves have sufficiently small amplitude, there exists a velocity potential $\Phi(\mathbf{x}, t)$ that satisfies both Laplace's equation and Bernoulli's equation in the fluid region. Linearizing Bernoulli's equation and a kinematic condition at the ice-water interface, and considering the balance of forces on the ice sheet, leads to an additional boundary condition to be satisfied there. To make the problem well-posed, it is also required that the transverse force and the bending moment vanish at the crack's open edges. The last 2 conditions are the natural boundary conditions for the free edges of an Euler-Bernoulli thin plate (Hilderbrand, 1965). In addition, there can be no flux through the seafloor, so the normal derivative Φ_z must vanish there.

Using the same nondimensionalization scheme as in Squire and Dixon (2000), and writing the velocity potential as:

$$\Phi(\mathbf{x}, t) = \mathrm{Re}[\phi(x, z)e^{i(ly-t)}], \tag{1}$$

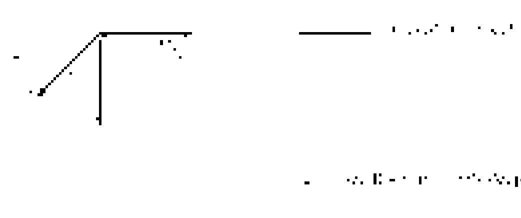

Fig. 1 Schematic diagram showing physical situation to be modelled

Received July 30, 2002; revised manuscript received by the editors May 21, 2002. The original version (prior to the final revised manuscript) was presented at the 12th International Offshore and Polar Engineering Conference (ISOPE-2002), Kyushu, Japan, May 26–31, 2002.
KEY WORDS: Crack, ice sheet, waves, Green's function.