International Journal of Offshore and Polar Engineering
Vol. 12, No. 1, March 2002 (ISSN 1053-5381)

Stationary and Hopf Bifurcations of Equilibrium Positions of a Cylinder Situated in Near and Far Wake Fields of an Upstream Cylinder

Wusheng Wu, Shan Huang and Nigel Barltrop*
Joint Department of Naval Architecture and Marine Engineering
Universities of Glasgow and Strathclyde, Glasgow, Scotland,
United Kingdom

ABSTRACT

This paper addresses the wake-induced instability of a pair of elastically mounted cylinders with one located in the other's wake. In particular, the effort is made to investigate the different instability mechanisms of the downstream cylinder in the near or far wake field of the upstream cylinder. It is found that the fluid-elastic instability of the downstream cylinder can be caused by either a Hopf or a stationary bifurcation, depending upon its position relative to the upstream cylinder.

INTRODUCTION

For a deepwater vertical riser cluster, one of the key design concerns is interference between the individual risers in strong ocean currents. The risers' lateral deflections are likely to be large, and the risers are prone to wake-induced clashing, with possible detrimental effects. In a series of papers published since 1993 on TLP/SPAR riser clearance in currents with a relatively large initial spacing between these risers, Huse (1993, 1996) proposed a simple yet remarkably accurate mathematical model for estimating time-averaged drag loading on the downstream riser situated in the wake of an upstream one. Huse also made a number of important experimental observations. In one experiment carried out to investigate the interaction between vertical risers, it was observed that, in addition to the high-frequency, vortex-induced response of amplitudes up to one-half of the diameter, the downstream riser also had low-frequency, in-line oscillations of an apparently very irregular nature, with the peak-to-peak stroke of these oscillations measuring 30 to 40 diameters or more. In the last few years, other researchers have contributed to the work (Furnes, 2000; Li and Morrison, 2000). These recent research contributions focus upon developing structural models to quantify impact loading on and possible damages to the risers.

Although relatively little work has been done on wake-induced, large-amplitude, low-frequency riser motions, there is a substantial amount of experimental and theoretical research on the fundamental problem of interaction between 2 rigid or flexible cylinders in currents; the research has applications to a variety of different engineering problems, such as power transmission lines, heat exchange tubes and chimneys. Broadly speaking, the prevalent approach of these works is 2-dimensional, with a relatively small spacing between the cylinders. The emphasis is largely on measuring the fluid loading on the downstream cylinder, which can be either stationary or vibrating, and on the effect of interference on vortex-induced vibrations. It is often assumed that the fluid flow responds to the cylinder's movement instantly, i.e. quasi-steady flow theory. The stability of the system is then analysed, often helped by utilising experimental data of the fluid forces available. Table 1 attempts to categorise these different applications. Most investigations focused upon their own engineering problems, with the parametric studies carried out within the ranges pertinent to the problem considered.

Based upon our previous work on this topic (Wu et al., 1999, 2001), a parametric investigation is carried out to examine the effects upon the downstream cylinder's stability of various, wide-ranging parameters. In doing so, for example, the different mechanisms of instability for power lines in air or marine risers in water can be clearly demonstrated. In particular, the effort is made to investigate the different instability mechanisms of a marine cylinder situated in the near or far wake field of an upstream cylinder. It should be noted that, as we focus upon the large-amplitude, low-frequency, instability-induced motions of the downstream cylinder, this paper considers only the time-averaged mean drag and lift forces on the cylinders.

THEORETICAL APPROACH AND RESULTS

Fig. 1 is a schematic diagram of the 2-cylinder system considered in this paper. The downstream cylinder is supported on 2 orthogonal springs, k_x and k_y, at a coupling angle of θ with respect to the flow direction. The origin of the coordinate system is at the centre of the upstream cylinder with x axis parallel to the incoming flow direction. X and Y are the streamwise and transverse relative distances between the 2 cylinders at their equilibrium positions. These are nondimensionlised by using the cylinder diameter.

In order to analyse the stability of the downstream cylinder, the position-dependent, fluid-force coefficients of the downstream cylinder have to be given beforehand. In this paper, the free-streamline method is applied to obtain the force coefficients over the majority of the wake field (Wu et al., 1999). In general, the force coefficients should be dependent upon the Reynolds number. However, it is known that in the sub-critical Reynolds number range, its effect is rather limited and the force coefficients are insensitive to its change. Wu et al. (2001b) give the detailed description of the stability analysis and its associated numerical calculations. The analysis is based upon the Routh-Hurwitz stability criterion. By utilising the criterion, it is possible to identify the critical state and the type of bifurcations when one of the eigenvalues passes through the imaginary axis. In addition, an extensive

*ISOPE Member.

Received July 18, 2001; revised manuscript received by the editors February 1, 2002. The original version was submitted directly to the Journal.
KEY WORDS: Marine riser, wake interference, stability, bifurcation.