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Numerical Examination of Third-order Wave Force on Axisymmetric Bodies

Bin Teng* and Guohai Dong
State Key Laboratory of Coastal and Offshore Engineering
Dalian University of Technology, Dalian, China

ABSTRACT

Within the frame of potential theory and the assumption of weak nonlinearity of wave motion, a numerical method is developed for the third-order triple-frequency wave loads on fixed axisymmetric bodies in monochromatic incident waves. Applying the numerical code, numerical computations were carried out for surge and heave forces and pitch moments on a uniform cylinder, truncated cylinders and a hemisphere. Examinations were made of the contribution to third-order forces and moments from potentials at each wave order, and the relation of third-order forces and moments to wave number and drafts of cylinders.

INTRODUCTION

It is believed that the third- or higher-order wave forces are the exciting sources for ringing responses of tension leg platforms (TLP) and gravity-based structures (GBS). Recently, a number of studies has been carried out for predicting third-order surge forces on cylinders. Based on the phenomenon that ringing occurs in long waves, Faltinsen, Newman and Vinje (1995) proposed a slender-cylinder theory. In such a long wave regime, the second-order diffraction can be neglected, and the third-order surge force is predicted by a kind of extension of the Morison equation. The attempt at full diffraction theory in the frequency domain was first made by Malenica and Molin (1995). They developed a semi-analytic solution for uniform cylinders in finite water depth, and successfully computed the third-order surge forces on them. Teng and Kato (1996) developed a numerical model for axisymmetric bodies by an integral equation method, which works for third-order surge forces, heave forces and pitch moments. Kim et al. (1998) developed a method for solving the full nonlinear wave diffraction from a uniform cylinder in the time domain. The third-order surge forces are then obtained by a Fourier analysis of the time history of full nonlinear wave forces. This method is also available for higher-order wave forces.

In this paper, the Teng and Kato method (1996) has been used to compute third- order triple-frequency surge force, heave force and pitch moment on cylinders and hemispheres. Examinations are made of the relation of the third-order forces and moments to body shapes, drafts of cylinders and wave frequencies, and the ratios of contributions to the third-order force and moments due to velocity potentials at each wave order. Numerical results show that:

- the third-order surge force on a truncated cylinder with a deep draft is close to that on the uniform cylinder with the same radius at the same water depth, and the third-order surge force on a hemisphere is smaller than that on the truncated cylinder with the same radius and draft;
- the third-order heave force on a hemisphere is larger than that on the truncated cylinder with the same radius and draft;
- the third-order pitch moment on the examined uniform cylinder has very big value at low frequency, and that on the truncated cylinder with a deep draft is close to that on the corresponding uniform cylinder at high frequency, but is very different at low frequency.

* ISOPE Member.

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KEY WORDS: Wave force, nonlinearity, axisymmetric bodies.

ANALYSIS METHOD

Free-Surface Condition

We consider the problem of wave diffraction from a vertical body of revolution, and define a right-handed coordinate system (x, y, z), with origin at the revolution axis of the body, $z = 0$ on the still, free surface and the z-axis pointing upward (Fig. 1). The fluid is assumed to be homogenous and incompressible, and the motion irrotational. There exists a velocity potential that satisfies the Laplace equation and the nonlinear free-surface boundary condition:

$$\Phi_{tt} + g\Phi_z + \frac{\partial}{\partial t} \, |\nabla\Phi|^2 + \frac{1}{2}\nabla\Phi\cdot\nabla|\nabla\Phi|^2 = 0 \qquad (1)$$

on the free surface $z = z(x, y, t)$, defined by:

$$\zeta = -\frac{1}{g}\Phi_t - \frac{1}{2g}\left[\nabla\Phi\cdot\nabla\Phi\right]. \qquad (2)$$

Under the assumption of weak nonlinearity, we can write the

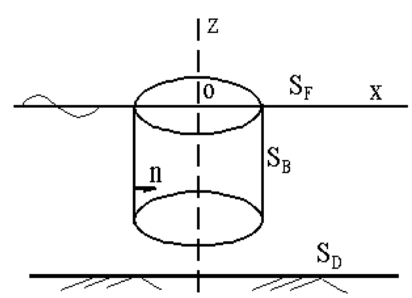

Fig. 1 Sketch of definition