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Reliability Analysis of Fatigue Damage Accumulation Under Variable Amplitude Loading

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ABSTRACT

Firstly, the Wirsching's model, which is widely employed in fatigue reliability analysis of marine and offshore structures, is analysed systematically. It is found that the very important random variable Δ in the Wirsching's model can not be directly determined from a fatigue experiment because of the irreversibility of the fatigue test. Secondly, by use of the statistical Miner's rule, a modified and also simplified Wirsching's model is proposed. Thirdly and more importantly, based on the two-dimensional probabilistic Miner's rule, a new model is established for fatigue reliability analysis of structural components subjected to specified cyclic loading of variable amplitude or stochastic time history. In the end, an example is presented, from which it will be seen that this new model is very convenient and feasible in practical engineering.

INTRODUCTION

Fatigue is one of the most serious failure modes of marine and offshore structures subjected to oscillatory environmental loads. There are two major sources of uncertainty in fatigue failure of structures: the stochastic nature of fatigue loading time-history, and the random variability of material resistance to fatigue loading. Therefore, it is necessary to develop a probabilistic-statistical theory of fatigue reliability in the analysis and design of engineering structures.

Generally, structural fatigue reliability theory consists of 3 parts: (1) the cycle counting technology of fatigue loading; (2) the reliability analysis of fatigue life and fatigue strength under constant amplitude loading; (3) the reliability analysis of fatigue life and fatigue strength under oscillatory loading of variable amplitude or stochastic time-history. Among all of those parts, the most key and tricky problem is how to establish the random fatigue accumulative damage rule. Since only after a reasonably random fatigue accumulative damage rule is established, does it become possible, in terms of constant amplitude experimental results, to perform the reliability-based analysis and design of structural components subjected to cyclic loading of variable amplitude or stochastic time-history?

At present, the Wirsching's model (Wirsching, 1980 and 1984; Wirsching and Haugen, 1973; Wirsching and Light, 1980; Martindale and Stahl, 1985; Knapp, 1985; Kumar and Kasan, 1990) concerning random fatigue damage accumulation is widely applied in marine and offshore engineering. In this paper, first, Wirsching's model is systematically reanalyzed; second, based on the statistical Miner's rule (Shimokawa and Tanaka, 1980), a modified and simplified Wirsching's model is presented; third, based on the two-dimensional probabilistic Miner's rule (Ni, 1994, Ni and Gao, 1996b), a new model is established of structural component subjected to a specified fatigue loading spectrum; in the end, an application example is presented and it shows that this new model is particularly relevant to predict fatigue life of structural components subjected to variable amplitude loading.

In the following, N_c denotes fatigue life under constant amplitude loading, N_v denotes fatigue life under variable amplitude or stochastic time-history loading, and N denotes the number of cycles of fatigue loading in any form.

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A RESTUDY OF WIRSCHING'S MODEL

The well-known Wirsching's model (Wirsching, 1984) can be summarized as follows. The first fundamental assumption is that the constant amplitude S-N_c curve takes the form:

$$N_c S^m = K \tag{1}$$

where S denotes stress range, hence $S = 2S_a$, and S_a denotes stress amplitude. In addition, it is implied that the mean stress $S_m = 0$.

Meanwhile, the second fundamental assumption is that the conventional Miner's rule is changed into:

$$\sum \frac{n_i}{N_{ci}} = \Delta \tag{2}$$

where Δ is regarded as a random variable.

Given a deterministic variable amplitude cyclic loading denoted as $(S_i, n_i | S_m = 0)$, $i = 1,2,\ldots,k$, from Eq. 1, one has $N_{ci} = K/S_i^m$, then it follows from Eq. 2 that:

$$D = \sum_i \frac{n_i}{N_{ci}} = \sum_i \frac{n_i S_i^m}{K} = \frac{N_v}{K} A\left(S^m\right) = \Delta \tag{3}$$

where $N_v = \sum_i n_i$, $A\left(S^m\right) = \sum_i \frac{n_i S_i^m}{N_v}$

therefore
$$N_v = \frac{\Delta K}{A\left(S^m\right)} \tag{4}$$

herein, both Δ and K are assumed as independent random variables, so N_v is also a random variable.