

# Wave-jet Nonlinear Interaction: Mathematical Model Development and Experimental Results

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**A 2DH model of coupled nonlinear interaction between regular waves and a velocity field is proposed. Wave effects on momentum equations are introduced by the divergence of the radiation stress tensor and the increase of the bottom friction factor. Kinematic conservation and wave action equations are written under a mild-slope hypothesis, accounting for diffraction and current effects. Wave breaking is taken into account by introducing a wave energy dissipation rate term, based on analogy to a turbulent periodic bore. A Doppler-shifted dispersion relationship and radiation stress tensor of the 1st-order Stokes wave theory are used. In order to highlight the nonlinearity of wave-current interaction, usually neglected, the proposed model has been tested against the results of wave flume experiments of interaction between regular waves and a free-surface jet. The observed jet spreading and velocity decay, enhanced by waves, are successfully reproduced by the model. Further, numerical simulations show that an uncoupled interaction model would lead to large errors in both wave and flow field predictions.**

## INTRODUCTION

Wave-current interaction plays an important role in several phenomena of interest to hydraulic engineers, such as sediment transport at river mouths and wastewater discharges into the sea, as well as the genesis of longshore currents, rip and undertow currents. A comprehensive study of all these phenomena is a challenging task, because it involves the simultaneous modeling of the current velocity field and wave field in the near-shore region, which is far from being established, especially in the surf zone.

The presence of velocity fluctuations due to waves indeed produces additional terms, the so-called wave stresses, in the time-averaged momentum balance equation of the flow field, in the same way that Reynolds stresses are due to higher-frequency turbulent velocity fluctuations (Longuet-Higgins and Stewart, 1964). In analogy with Reynolds stresses, a closure problem arises also for wave stresses calculation. By assuming waves following some existing wave theory, analytical expressions for the wave stresses may be derived. At the same time, current velocity affects wave refraction and diffraction, giving rise to a Doppler shift of phase and group speed and to additional terms in wave number and wave energy equations.

A number of experimental observations confirms the importance of such nonlinear interaction. Wave shoaling is affected by the presence of a jet and, at the same time, the jet velocity field changes, since jet spreading is enhanced by momentum exchange with opposing waves (Ismail and Wiegel, 1983; Koole and Swan, 1994; Di Natale and Vicinanza, 2001). So far, few fully coupled models of mutual interaction between waves and currents have been proposed (Yamaguchi, 1986; Park and Borthwick, 2001), while most models address only aspects of wave-current interaction.

Some models deal only with the effects of assigned currents on the wave field, such as wave transformation in the presence of uniform currents parallel to waves (Jonsson and Arneborg, 1995; Pihl et al., 2001); wave shoaling in the presence of currents with a nonzero vorticity profile (Thomas, 1990; Swan and James, 2001); wave refraction-diffraction models in the presence of a priori given velocity fields (Booij, 1981; Kirby, 1986; Teng et al., 2001; Holthuijsen et al., 2003).

On the other hand, other models calculate the velocity field induced by assigned waves: longshore current models (Longuet-Higgins, 1970; Kraus and Sasaki, 1979; Thornton and Guza, 1986; van Rijn and Wijnberg, 1996; Li and Johns, 1998; Ruggiero and McDougal, 2001); rip current models (Wind and Vreugdenhil, 1986); undertow models (Svendsen, 1984; Stive and Wind, 1986; Wind and Vreugdenhil, 1986; Veeramony and Svendsen, 2000); near-shore circulation models (Rakha and Kamphuis, 1997); and models of jet spreading in the presence of opposing waves (Ismail and Wiegel, 1983; Yoon and Liu, 1989; Di Natale and Greco, 2003).

This paper introduces a 2DH model of fully coupled mutual nonlinear interaction between surface waves and currents. The flow field model consists of continuity and momentum equations, while the wave refraction-diffraction model, based on the Doppler-shifted 1st-order Stokes dispersion relation, consists of the wave action balance equation and kinematic conservation equation, both modified to take into account wave interaction with the current.

Model equations are given below. The model is numerically integrated by means of a semi-implicit finite-differences scheme. Simulations are carried out in order to compare model predictions with the experimental results of the jet current from wave flume experiments.

## MATHEMATICAL MODEL DESCRIPTION

The proposed mathematical model of coupled waves and current interaction is based on the assumption that instantaneous velocity fluctuations from time-averaged value may be split into waves-due oscillations and turbulent fluctuations (Longuet-Higgins and Stewart, 1964). The shallow water depth-averaged

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**KEY WORDS:** Wave-jet interaction, 2DH model, wave diffraction, radiation stress tensor.