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Wave Response Analysis of a VLFS by Accelerated Green's Function Method in Infinite Water Depth

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This paper presents an accelerated Green's function method for wave response analysis of a VLFS (Very Large Floating Structure) in infinite water depth. The formulations have been made for multipole expansion of the Green's function in infinite water depth and the expressions for conversion of the expansion coefficients, which are required for application of the fast multipole algorithm. Then the paper presents wave response analysis for a VLFS in infinite water depth using an accelerated Green's function method. Numerical examples have shown the efficiency of $O(N)$ in computation time, where N is the number of unknowns for the velocity potential.

INTRODUCTION

The Boundary Element Method (BEM) using the water wave Green's function (Green's function method hereafter) has been used as a powerful tool for wave diffraction/radiation analysis for a large offshore structure. Green's function method requires $O(N^2)$ storage and $O(N^3)$ CPU time when a direct solver is used, or $O(N^2)$ CPU time even when an iterative solver is used, where N is the number of unknowns. Thus it had been difficult to apply the conventional Green's function method for wave diffraction/radiation analysis for a Very Large Floating Structure (VLFS) such as a floating airport because of the huge storage and CPU time requirement.

Utsunomiya et al. (2001, 2002, 2003) have thus developed the accelerated Green's function method, where the fast multipole algorithm (Greengaard, 1988) has been applied to the water wave Green's function method. Using the accelerated Green's function method, the storage and CPU time requirements have been reduced to $O(N)$. The hydroelastic analysis of a VLFS in variable water depth (Utsunomiya et al., 2001, 2002) and that of a hybrid-type VLFS (Utsunomiya et al., 2003) have also been demonstrated. However, because the method utilizes the series form of the Green's function in finite-depth waters, the method becomes ineffective if the water depth becomes large relative to the horizontal dimensions of the floating structure. This is because the number of terms in the series form of the Green's function must be taken to be larger as the water becomes deeper.

The objective of this study is then to develop the accelerated Green's function method in infinite water depth, where the integral form Green's function is used instead of the series form. The main body of the paper first presents necessary formulations for application of the fast multipole algorithm. Wave diffraction analysis around a VLFS is then shown, and finally an example of the wave response analysis for a hybrid-type VLFS is demonstrated.

FORMULATIONS

Fast Multipole Algorithm

The basic idea of the fast multipole algorithm is to evaluate the potential from far-field as a group, as schematically shown in Fig. 1. (For details, see Greengaard, 1988.) All boundary elements (or panels) are organized into a tree-structure by introducing cells to which a number of elements belongs. The algorithm consists of the 2 major paths, that is, the upward and the downward paths.

Upward path. In this, the multipole expansion coefficients are first to be calculated for all elements (or panels) which belong to a leaf cell, and by summing up the coefficients at the center of the leaf cell, we obtain the multipole expansion coefficients for the leaf cell. This algorithm is referred to as MP. Next, by moving the multipole expansion point of the leaf cell to the center of the parent cell and by gathering all of the multipole expansion coefficients of the child cells, we obtain the multipole expansion coefficients for the parent cell. (This is referred to as M2M.) By repeating this process for all cells in the upward direction, we obtain the multipole expansion coefficients for all cells (from leaf cells to the upper large cells.)

Downward path. In this, the following 2 kinds of calculations are to be made.

- If the parent cell has local expansion coefficients, by moving the local expansion points to the center of its own cell, we obtain the local expansion coefficients of the specified cell (referred to as L2L).

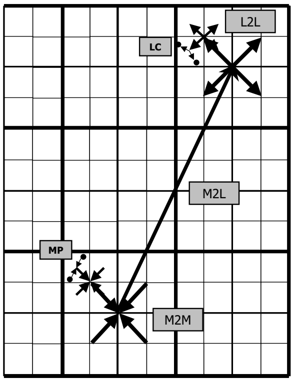

Fig. 1 Schematic representation of fast multipole algorithm

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KEY WORDS: VLFS, Very Large Floating Structure, Green's function method, fast multipole method, infinite water depth, potential theory, boundary element method.