International Journal of Offshore and Polar Engineering
Vol. 6, No. 2, June 1996 (ISSN 1053-5381)

A New Stochastic Approach for Estimating Extreme Statistics of Slow Drift Motion of Moored Floating Structures

Shunji Kato
Ship Research Institute, Ministry of Transport, Tokyo, Japan

Takashi Okasaki
The Institute of Statistical Mathematics, Ministry of Culture and Education, Tokyo, Japan

ABSTRACT

A stochastic theory for estimating the response statistics of moored floating structures subjected to slow drift forces is presented using an approximate solution of the Generalised Fokker Planck equation.

INTRODUCTION

The statistical theory of wave drift response of moored floating structures in random seas has been developed gradually over the last two decades. It can roughly be divided into two main categories: Methods based on Gaussian and quadratic systems; methods based on modelling the response as a Markov vector. The first method is to represent the wave drift response as a quadratic (Volterra) functional transformation to Gaussian excitation. One of the first efforts to describe the probability distribution of the wave drift response was presented by Neal (1974). Following this seminal contribution, numerous efforts have supplemented and extended Neal's first result. Notable contributions have been given by Vinje (1983), Naess (1985) and Kato et al. (1990). Few attempts have been made in the literature to account for the non-Gaussian nature of the wave drift response process when predicting extreme values, e.g. the asymptotic approach (Naess, 1985), the approximate method based on the assumption of the displacement-velocity independence (Kato et al., 1987) and the method using a series form on the joint probability function (MacWilliam and Langley, 1993). However, the nonlinear nature in the equation of wave drift motion and the effect of correlation between maxima on extreme response are not taken into account enough.

While the second method is to model the response as continuous Markov processes, in this approach, random excitation is approximated as a white noise process.

Within the limitation of the modelling, which essentially approximates the actual excitation processes as white noises, the theoretical framework of Markov process theory offers, in principle, a direct approach to the exact treatment of nonlinear random dynamic problems, which are difficult to tackle by other methods.

A special feature with the Markov vector processes is that their statistical properties, i.e., joint probability density functions, are determined by solving a partial differential equation usually denoted as the diffusion equation or the Ordinary Fokker Planck (OFP) equation. The OFP equation fails, however, to determine the joint PDF correctly, except that the excitation process is close to a Gaussian white noise process, because it has no term incorporating the detailed information about the excitation but a single diffusion coefficient.

For the case where the excitation can not be adequately approximated as the white noise process, the excitation may itself be modelled as a filtered white noise or response of a dynamic system driven by white noise. In this case, the multidimensional FP (MFP) equation for the joint PDF of the response and excitation variables is derived.

Several attempts have been made to solve the MFP equation. A very common approach is to express the solution as a series of trial functions. The unknown coefficients in the series are determined from the diffusion equations, for instance, by using a weighted residual method (Marthinsen, 1987). However, higher-order coefficients (moments) are generally difficult to obtain. This, together with the fact that with respect to extreme value estimates these series converge very slowly, makes the practical use of this method limited.

The stochastic average method, which aims at reducing the dimension of the problem, has been in use for some time (Roberts, 1986). This method is restricted to lightly damped systems.

In recent years there has been a significant effort towards developing a numerical direct solution method of the MFP equations.

Johnsen and Naess (1991) exploited a numerical scheme that it is possible to construct an approximate transition probability density also for nonlinear slow drift response problems. This solution procedure is often denoted as the path integral method. The method is promising, and it is shown that it makes it possible to obtain solutions at low probability levels relevant to extreme value estimation. However, since this method is, in principle, based on numerical procedures, it is impossible to investigate analytically how the response PDF changes together with the intensity of the non-Gaussian excitation process and the damping and stiffness coefficients in the equation of motion.

In contrast, there is a small attempt to generalise the OFP equation to be applicable to non-Gaussian excitation. Stratonovich (1967) considered a general stochastic equation with noise of arbitrary statistical properties, and obtained an equation governing the PDF in a form of functional series. Picking up the first and

Received March 8, 1995: revised manuscript received by the editors March 18, 1996. The original version (prior to final revised manuscript) was presented at the Fifth International Offshore and Polar Engineering Conference (ISOPE-95), The Hague, The Netherlands, June 11-16, 1995.

KEY WORDS: Non-Gaussian excitation, renormalization, perturbation, Generalized Fokker Planck equation, stochastic process, slow drift motion.